Alpha, Alpha, Whose got the Alpha?

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Introduction

At recent alternative investment seminars and conferences, hedge fund manager after hedge fund manager remains intent on proving their ability to produce something they refer to as 'alpha'. Each manager and investor has his or her own unique take on what alpha is or how it should be measured. It should come of no surprise that academics have weighed in on the central questions of this issue; what is alpha, and what is the best way to measure the alpha of an investment strategy?

The term alpha derives from statistics. In linear regression, the equation that relates an observed variable y to some other factor x is written as

$$y = a + b\,x + \varepsilon.$$

The first term, a (alpha) represents the intercept, b (beta) represents the slope, and e (epsilon) represents a random error term. In finance, we generally assume that returns to some asset have a linear relationship to the returns to one or more factors or performance benchmarks. The alpha term is important in finance because it represents the return that the investor would receive if the benchmark had a zero return. As such, it is a proxy for manager skill. Rearranging the previous formula (and ignoring the error term for now), we can restate the equation to focus on the alpha:

$$a = y - b\,x.$$

We also prefer to think about returns that are net of the risk-free rate, so we adjust both the asset and the benchmark returns downward by the risk-free rate of interest: Using the following definitions,

R_i = Return on Fund i
R_f = Return on a risk-free asset such as Treasury bills
R_m = Factor or benchmark such as the SP500, MSCI, or MAR CTA Index, the equation for alpha becomes:

$$a = (R_i - R_f) - b\,(R_m - R_f)$$

While this equation is preferred by the academic community, there are a number of variations on the theme. The following are a few of the definitions of alpha presented at a recent alternative investment conference.

1) $a = R_i - R_f$

2) $a = \mathrm{R}_i\text{-}\mathrm{R}_m$

3) $a = R_i - b\ R_m$, where b is estimated using historical data in a simple regression (OLS) model: $b = \mathrm{Corr}(R_i, R_m) \text{ X } Stdev(R_i) / Stdev(R_m)$ over some historical time period.

4) $a = (R_i - R_f) - b\ (R_m - R_f)$ where b is estimated as in (3)

The above are all correct under very limited circumstances. It is not the purpose of this article to give a complete statistical and theoretical review of each of the above or to attempt to re-educate the entire investment community brought up on the mother's milk of Modern Portfolio Theory and the Capital Asset Pricing Model.

In the world of academics, alpha is generally defined as the excess return to active management, adjusted for risk. It is the return adjusted for the risk of a comparable risky asset position or portfolio. The question is therefore how to define the expected risk of the manager's investment position and how to obtain the return on a comparable risk position or portfolio. Before exploring various approaches to alpha determination for alternative investments, a few points about measurement of investment performance should be reviewed:

1) Modern Portfolio Theory (first presented in 1952, it is ancient portfolio theory now) has nothing to say about alpha or how it should be measured.

2) The CAPM is a theoretical concept first derived in the early 1960's. It is a dated concept. We now know that it is difficult to prove conclusively since we have no idea what the true market portfolio is and therefore what a security's true beta is, even if you believed all the assumptions of the CAPM.

3) The Arbitrage Pricing Theory (APT) is also twenty years old. It represented an alternative expected return model for those who felt the CAPM too restrictive. After almost twenty years of review, we know that the APT, which focuses on comparing the return of two like assets sensitive to the same market-wide factors, is likewise testable only under the most severe restrictions.

The failure of any single theoretical model to describe the expected return process in a testable form leaves us with statistical, rather than theoretical, approaches to estimate a security's or strategy's expected return.

It is not appropriate to say that you have a positive alpha (net risk-adjusted return) simply because the return it is greater than the risk free rate unless your portfolio is risk-free. Similarly, comparing return to the S&P 500 or any other benchmark is inappropriate unless your strategy responds only to the same return drivers that drive the S&P 500 or the cited benchmark. A quick review of the above approaches to alpha may help clarify the point.

1) $a = R_i - R_f$, *assumes* $b = 0$

The risk free rate is the return benchmark only if your return is not affected by any systematic information or associated market return factors other than that implied in the risk free rate. For example, a market-neutral portfolio (relative to the S&P) should never derive its alpha relative to the risk free rate. Use of the risk-free rate assumes a zero beta. However, we know now that the CAPM equation is not testable and that the S&P 500 is not the market portfolio. As a result, a portfolio that is not correlated with the S&P 500 (and therefore has a zero S&P 500 beta) may still have a high variance relative to the risk free rate and still be sensitive to a host of market-related systematic risk factors (e.g., interest rates). For those interested in academic theory, in a subsequent version of the CAPM, the zero-beta CAPM, the expected return on the zero-beta portfolio is also greater than the risk free rate.

2) $a = R_i - R_m$, *assumes* $b = 1$

This common variation ("My strategy beat the Lehman Bond index by 4% last year") assumes that the reference benchmark is the appropriate benchmark *and* that the strategy has the same leverage as the benchmark. With the exception of a strategy that is designed to replicate the returns of the benchmark, the alpha generated by this approach is essentially meaningless.

3) $a = R_i - b\, R_m$, *assumes* $R_f = 0$

A simple adjustment to equation (2) is often made in order to adjust for the beta of the portfolio with respect to the benchmark. This is better than (2), but should be used only if your return is not affected by any systematic information or associated market return factors other than that implied in the comparison return benchmark. For example, a domestic opportunity portfolio with a small-firm bias should never derive its alpha relative to the S&P 500. It should use (if possible) a series of passive indices (adjusted for risk and leverage) which replicate, *ex ante*, the risky positions in their portfolio. The main problem with (3) is that it uses nominal, rather than excess returns. By assuming the risk-free rate is zero, an asset with higher leverage than the benchmark will appear to have a positive alpha even if it does not. A positive alpha from equation 3 does not imply superior performance. The alpha must be greater than $(1-b)R_f$ in order to imply a positive risk-adjusted return.

4) $a = (R_i - R_f) - b\,(R_m - R_f)$

Equation 4 is the preferred method of estimating alpha, but a number of potential problems arise when trying to estimate the correct alpha. Historical data must be used over some period. This introduces the possibility of measurement error as well as selection bias in the managers choice of the time period to be used. Outliers in the data may lead to strategies with measured low (high) betas relative to the true beta simply due to known biases in the method OLS uses to fit the regression line.

Many hedge fund strategies have a low measured beta relative to the S&P 500 over long periods of time, but over shorter periods may have a high beta. In addition, use of a single-index model assumes that the market factor in the single index model replicates the fundamental risk factor driving the return of the strategy. If not, a multi-factor model should be used to describe the various market factors that drive the return strategy. One of the basic tenets of statistical regression says it is better to over-specify a model (include more sources of systematic risk than the fund is exposed to) than under-specify (include fewer factors). If the model is over-specified, many of the betas will simply be zero. However, if under-specified, there is the possibility of significant bias.

Sharpe Ratio

None of this discussion mentioned the Sharpe ratio (the excess rate of return divided by the standard deviation, $(R_i\text{-}R_f)/Stdev$). The Sharpe ratio is a common performance measure, and it is often asserted that a high Sharpe ratio implies management skill. However, the ratio is only loosely related to alpha.

The Sharpe ratio is a stand-alone measure, while investors are primarily concerned with the question of whether an asset should be added to an existing portfolio. Modern (or even ancient) portfolio theory tried to show that variance was a poor measure of comparison risk if the two assets were to be held in a portfolio. In that case one should consider the risk of an asset as its marginal contribution to the risk of the investor's portfolio. Break-even analysis is often used to test for the potential contribution of an asset to the risk/return profile of an existing stand-alone portfolio. The break-even (R_c) and excess break-even rate of return (EBK) is often computed as follows:

$$R_c = \left[\left(\frac{R_p - R_F}{\sigma_P}\right) P_{CP}\right] \sigma_C + R_F \qquad\qquad EBK = R_c - \left[\left(\frac{R_p - R_F}{\sigma_P}\right) P_{CP}\right] \sigma_C + R_F$$

where,

> R_c = Break-even rate of return required for CTA index to improve Sharpe ratio of the alternative index *p*
> R_f = Riskless rate of return
> R_p = Rate of return on alternative index p
> P_{cp} = Correlation coefficient between CTA index c and alternative benchmark index p
> σ_c = Standard deviation of CTA index c
> σ_p = Standard deviation of alternative index p

Some alternative investment managers emphasize the non-correlation of their strategy with the S&P 500 and then turn around and offer a comparison of their Sharpe ratio with that of the S&P 500 to indicate superior alpha performance. This may be suitable only if standard deviation is the sole and proper measure of risk (which it may not be). Even in this case this comparison will not indicate its potential alpha benefit relative to other, non-tested, active manager portfolios nor does it provide an indication if another similar investment will have provided a similar or even greater increase in the Sharpe ratio. Moreover, given the zero correlation with the S&P 500, the relative Sharpe ratio of a strategy with the S&P 500 may not even indicate the relative benefit of adding that strategy relative to the S&P 500 to another portfolio. For instance, it is possible that a manager's strategy may have a zero correlation with the S&P 500, but a higher correlation than the S&P 500 with another existing stand-alone portfolio and therefore a higher required breakeven return than the S&P 500.

One of the problems in the use of simple stand-alone performance comparison measures is that hedge funds, private equity, managed futures are often called 'alternative assets' in contrast to 'additional assets'. The name alternative implies that it is either this or that; that is, investment in the S&P 500, Lehman bond index etc. or the hedge fund strategy. Nothing could be farther from the truth. They are different investments offering different returns in different market environments. Alternative asset strategies relative to long only stock and bond investments should be considered relative to the potential relative benefits that provide via diversification to one's existing portfolio. The ability of a manager to achieve alpha is based on his or her ability to achieve a return via an active strategy that is greater than that one could achieve using a passive strategy designed to capture the same risks and hence the same expected returns of the active strategy. If that strategy fits into the existing portfolio of an investor and helps that investor achieve his or her unique goals, it should be added to one's portfolio as an additional investment in contrast to a similar passive strategy.

Alice in Alice in Wonderland asked the Cheshire cat what path to take. The cat answered where she wanted to go. Alice replied that she had no idea. The cat responded, then it really doesn't matter which path you take. Managers must know which path they wish to take; that is, alpha as a marketing devise or as a measure of comparable risk/return performance. If managers wish to define alpha to fit their own marketing purpose and use alpha to sell a product, it is understandable. However, one should never mistake a 'marketing' alpha from a relative-performance alpha. If the manager can chose asset positions with a higher return (but the same *ex ante* risk) to some comparable naïve investment position then that person can be said to achieve a positive alpha. Managers may say that investors' never care about relative return, but only absolute return. But performance alpha is all about properly measured relative return. Unfortunately, we have no simple method for establishing this benchmark except under vary restrictive situations. However, at least we do know that because any investment decision involves some risk, the riskless rate is probably not appropriate as a benchmark. How much return should be added and what method should be used to determine the incremental return to add to the risk free rate to obtain the appropriate return comparison is still open for discussion. Despite the difficulty it is an attempt worth the effort.