Fat tail risk in portfolios of hedge funds and traditional investments

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Abstract:

We analyse the risk of portfolios mixing hedge funds, stocks and bonds. The risk of the portfolios is quantified by the Value-at-Risk and the Expected Shortfall derived from the Extreme Value Theory. This approach enables us to take the impact of higher moments into account. We show that the risk of a traditional portfolio is reduced by the inclusion of hedge funds. An optimal weight of 50% hedge funds is found when the traditional portfolio is mostly composed of bonds. In equity dominated portfolios, investors should incorporate as much hedge funds as possible. Furthermore we examine the extremal dependence between funds of hedge funds and stocks or bonds using multivariate Extreme Value Theory. We do not find any significant extremal dependence between hedge funds and bonds. The evidence is more mixed between stocks and funds of hedge funds. Funds of hedge funds without a significant investment in Managed Futures exhibit significant dependence in the extreme with the stock market. The August 1998 event linked to the Russian crisis and the LTCM failure is the cause of this dependence.

Keywords: hedge funds, higher moments, extreme value theory, VaR, Expected shortfall

JEL code: C13, C22, G11.

1 Introduction

In the last decade, hedge funds have been the fastest growing asset class of the financial sector. According to *TASS Research*, the total net assets in single manager hedge funds are estimated to be approximately 800 billions USD[2] as third quarter 2003. The attractiveness of hedge funds may be explained by their good performance associated with low volatility and low correlation to traditional investments. Indeed, hedge funds are known in the financial community to present "bond like" volatility. According to McFall Lamm (1999), hedge funds should even replace bonds in the portfolio of investors.

However, recent evidence (see e.g. Brooks and Kat (2002), Schmidhuber and Moix (2001)) cast doubts on the validity of the volatility as the risk measure for hedge funds. Indeed, the returns of hedge fund indices are not normally distributed exhibiting unusual levels of skewness and kurtosis. These characteristics are consistent with the complex trading strategies used by hedge funds which present option-like payoffs. As a consequence, the analysis of hedge funds based solely on mean and variance may be leading to wrong conclusions and decisions (see Bacmann and Scholz (2003) in the context of performance measurement and Bacmann and Pache (2003) in the context of portfolio optimisation). In a recent paper, Amin and Kat (2003) show that while hedge funds combine well with stocks and bonds in the mean-variance framework, this is no longer the case when skewness is considered.

The preferences of risk averse investors with respect to the different moments of the return distribution[3] (see Scott and Horvath (1980) and Pratt and Zeckhauser (1987)) imply a trade-off between these moments. In other words, an investor may accept a more negative skewness if the variance is reduced at the same. Thus, the preferences of investors have strong implications for the analysis of risk in the hedge fund world. In particular, analysing individually the different moments, as in Amin and Kat (2003), impede from measuring the substitution effects between the moments and their implications on the portfolio allocation decision.

In this paper, we advocate the use of the Extreme Value Theory (EVT)[4] in order to take the behaviour of extreme events into account. This area of statistics enables the estimation of the Value-at-Risk and the Expected Shortfall under fairly general types of distributions. VaR and ES

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[2] This number includes the assets run by hedge fund managers in private managed accounts.

[3] Risk averse investors tend to like positive skewness and to dislike high level of kurtosis

[4] Although EVT is only recently used in financial application, it can be traced back to the early 1920s.

estimated via EVT do not have to describe explicitly the substitution between the different moments as these measures rely on the standard deviation of the distribution as well as on a shape parameter describing the left tail of the distribution. The substitution effects are treated as endogenous.

This paper contributes to the growing literature on the risk associated to hedge funds in two main directions. Firstly, it carefully examines the risk of portfolios built with stocks, bonds and hedge funds using the Extreme Value Theory. Several papers (Blum, Dacorogna and Jaeger (2003), Gupta and Liang (2003), Lhabitant (2003)) have already been using the Value-at-Risk derived from EVT in the context of single funds or hedge fund indices. However, none of them has been exploring the risk properties of portfolios including stocks, bonds and hedge funds. Secondly, we measure the dependence in period of crisis between hedge funds and traditional investments. For that purpose, we test explicitly the existence of asymptotic dependence between hedge funds and traditional investments (stocks or bonds).

Our paper is organised as follows. Section 2 reviews the implications of univariate and multivariate extreme value theory in order to estimate the risk of portfolios including hedge funds. In section 3, we empirically examine and compare the risk of hedge funds and traditional investments. In section 4, we analyse the risk of portfolios built out of hedge funds, stocks and bonds. Moreover, we explicitly measure extreme dependences between hedge fund and traditional investments. Section 5 concludes this article.

2 Measuring the risk of hedge funds

2.1 How to measure risk

The definition of risk is a particularly difficult task as no common acceptance exists. In the financial community, risk is usually related to the uncertainty of the future outcome of a decision today. The different possible outcomes are linked to specific probabilities. Analysing the whole range of probabilities, also called probability distribution, is not tractable in practice. This is why simple statistical measures are used to assess the magnitude of risk. The standard deviation, also known as volatility, has been the most widely used measure to achieve this task. However, this measure relies on the assumption that the return distribution is symmetric around its mean and implies that the sensitivity of the investor is the same on the upside as on the downside. This very strong assumption has been challenged by the emergence of the prospect theory (see Kahnemann and Tversky (1979) for more details on the prospect theory). In this framework, the investor is more affected by a drop in his wealth than by an increase.

In order to take the asymmetry in the return distribution as well as in the investor preferences into account, the use of downside deviation as risk measure has been frequently advocated (see e.g. Sortino and Price (1994), Bacmann and Pache (2003)). However, even though this measure is sensitive to extreme events of the return distribution, it does not provide a full characterisation of these extreme events as defined by the extreme quantiles of the distribution. In such a context, the Value-at-Risk, designed to capture the maximum loss over a target time horizon with a given degree of confidence, is far better suited. VaR has been getting a very wide acceptance throughout the financial community as it translates a complex risk notion into a simple and synthetic monetary amount[5].

The major drawback of VaR is that it considers only one particular point of the distribution. Indeed, no information is given when the loss exceeds the VaR level. Moreover, as pointed out by Artzner et al. (1999), VaR is not a "coherent risk measure" as under certain circumstances, the VaR framework neglects the diversification effects. A risk measure, known as conditional VaR or expected shortfall, has been designed to address this issue. It quantifies the expected loss given the fact that the loss will be exceeding the VaR level. Contrary to VaR, the expected shortfall qualifies as a coherent risk measure.

The estimation of VaR and of ES relies on three different variables: the target horizon, the confidence level and the estimation model. Following Gupta and Liang (2003), we set the target horizon to one month. The choice of the estimation model for VaR and the expected shortfall is critical in the hedge fund context. The estimation of VaR based on the log normality assumption of prices as proposed by Riskmetrics is not applicable to hedge funds which exhibit fat tailed and asymmetric returns. In order to capture the extreme tail better than the standard VaR calculation, Li (1999) proposes to estimate VaR based on the volatility, skewness and kurtosis. Signer and Favre (2002) and Favre and Galeano (2002) introduce a similar concept with the modified VaR based on a Cornish-Fisher expansion. However, this kind of approach suffers from an important drawback. It assumes that the first four moments do exist. As pointed out by Dacorogna et al. (2001), the convergence of the fourth moment is not guaranteed for

[5] VaR serves as the basis for regulatory requirements in terms of capital adequacy in banks and financial institutions. In the context of hedge funds, Gupta and Liang (2003) report that most hedge funds are well funded according to VaR and to the guidelines from the Basel Committee on Banking Supervision (1996).

financial data. In other words, quantities can always be computed but cannot be used to obtain a reliable estimate of VaR[6]. As a consequence, VaR and expected shortfall should be estimated via a more reliable theory, namely the Extreme Value Theory.

2.2 Extreme value theory

The Extreme Value Theory (EVT) provides a very powerful tool in order to analyse the risk in the extremes. In this section, we briefly review the main aspects of EVT. A more detailed and comprehensive description of this theory can be found in Embrechts, Klüppelberg and Mikosch (1997) and Focardi and Fabozzi (2003). Two different approaches have been employed in the EVT framework: the block maxima method and the peak over threshold method.

The block maxima method focuses on the limiting distribution of the maximum of a sequence of independent, identically distributed random variables with common distribution F. It can be shown that this limiting distribution is a non-degenerate distribution function H. From the so-called *extremal type theorem* it is known that H must be one of the three fundamental types[7] of extreme value limit laws:

Type I (Gumbel or thin-tailed class)): $\Lambda(x) = \exp\{-e^{-x}\}, -\infty < x < \infty,$

Type II (Fréchet or heavy-tailed class): $\Phi_a(x) = \begin{cases} 0, & x \le 0 \\ \exp\{-x^{-g}\}, & x > 0 \end{cases} \quad g > 0,$

Type III (Weibull or short-tailed class)): $\Psi_a(x) = \begin{cases} \exp\{-(-x)^{g}\}, & x \le 0 \\ 1, & x > 0. \end{cases} \quad g < 0,$

In practice, the block maxima method suffers from an important drawback. Within this method the maximum value is computed over a block or sub-period of a given size n. The series of obtained block maxima is fitted to one of the extreme value laws. The blocks are defined as non overlapping periods of equal length n and the period is selected so that it represents year, season, month or any other calendar item. However, there is no consensus on the size of the blocks. Moreover, this method is very demanding in terms of data.

Rather than dividing the data set into sub-periods and selecting the maximum value in each sub-sample, the peak over threshold (POT) method offers a more efficient use of the data set,

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[6] The expected shortfall cannot be derived within the Cornish-Fisher expansion framework.

[7] The three types can be combined into the single *Generalised Extreme Value* distribution (GEV)

where all values exceeding a (high) pre-specified threshold have to be considered. The theoretical background of this methodology was derived by Pickands (1975). Conditional on the event that the random variable X is larger than the threshold u and denoting these exceedances by Y, we can define the distribution function F_u called the *conditional excess distribution function* as

$F_u(y) = P(X - u \le y \mid X > u), \qquad 0 \le y \le x_F - u$ where $x_F \le \infty$ is the right endpoint of F.

Pickands (1975) shows that the *Generalised Pareto distribution* (GPD) is the limiting distribution for the distribution of the excesses, $F_u(y) \approx GPD_{x,s}(y)$, for $u \to \infty$. The GPD is defined as

$$GPD_{x,s}(y) = \begin{Bmatrix} 1 - \left(1 + \frac{x}{s} y\right)^{-1/x} \\ 1 - e^{-y/s} \end{Bmatrix} \text{ if } \begin{cases} x \neq 0 \\ x = 0. \end{cases}$$

with scale and shape parameters s and x respectively. The mean of this distribution exists if $x < 1$ and the variance if $x < 1/2$; more generally, the *k*th moment exists if $x < 1/k$.

Redefining the GDP as a function of x with $x = u + y$, i.e. $GPD_{x,s}(y) = GPD_{x,u,s}(x)$, the following model can be derived to build a tail estimate of $F(x)$:

$\hat{F}(x) = (1 - F(u)) GPD_{x,u,s}(x) + F(u)$

F_u is now replaced by GPD and the $F(u)$ can be estimated by $(n - n_u)/n$, where n is the total number of observations and n_u the number of observations exceeding the threshold u.

This translates into $\hat{F}(x) = 1 - \frac{n_u}{n}\left(1 + \hat{x}\frac{x-u}{\hat{s}}\right)^{-1/\hat{x}}$.

VaR is obtained by inverting for a given probability a :

$$\hat{VaR}_{1-a} = u + \frac{\hat{s}}{\hat{x}}\left(\left(\frac{n}{n_u}(1-a)\right)^{-\hat{x}} - 1\right).$$

The derivation of the formula for the expected shortfall is straightforward as $ES_{1-a} = VaR_{1-a} + E\left[X - VaR_{1-a} \middle| X > VaR_{1-a}\right]$. It can be shown that the estimator for the expected shortfall is:

$$\hat{ES}_{1-a} = \frac{\hat{VaR}_{1-a}}{1-\hat{x}} + \frac{\hat{s} - \hat{x}u}{1-\hat{x}}.$$

2.3 Extreme dependences

Measuring the risk of individual asset classes is usually not sufficient. More insight can be gained by analysing the dependences between the different classes. However, measures like the correlation is not applicable when returns are not normally distributed (see e.g. Kat (2003)). In this section, we present how to measure extreme dependences between asset classes. To our knowledge, only one paper (Blum, Dacorogna and Jaeger (2003)) has tried to measure the dependences between hedge funds and traditional indices. However, this paper assumes that the bivariate distributions are well described by elliptical distributions. We believe that this assumption is too strong as the distributions in the hedge fund world are usually asymmetric. As a consequence, we apply the concept of asymptotical dependence (see e.g. Ledford and Tawn (1996, 1997)). More specifically, we test the existence of asymptotical dependences between the different asset classes, as suggested by Poon, Rockinger and Tawn (2003).

Following Poon, Rockinger and Tawn (2003), we transform the bivariate returns (X,Y) to unit Fréchet marginals (S,T). This transformation removes the influence of the marginal aspects of the initial random variables while keeping the differences due to dependences aspects. More specifically, the transformation can be expressed as:

$S = -1/\log F_X(X)$ and $T = -1/\log F_Y(Y)$

where F_X and F_Y are the empirical marginal distribution functions for X and Y.

Two different types of bivariate distribution can be identified depending on the dependence coefficient value. This coefficient is defined as follows:

$$c = \lim_{s\to\infty} \Pr(T > s | S > s)$$

where $0 \le c \le 1$. The random variables are said to be asymptotically dependent if $c > 0$ and asymptotically independent if $c = 0$.

When the two variables are asymptotically independent (i.e. $c = 0$), the coefficient c is not sufficient to describe the dependence of the two variables. This is why Coles, Heffernan and Tawn (1999) advocate the use of $\bar{c}$ defined as:

$$\bar{c} = \lim_{s \to \infty} \frac{2 \log \Pr(S > s)}{\log \Pr(S > s, T > s)} - 1$$

where $-1 < \bar{c} \leq 1$. This quantity measures the rate at which $\Pr(T > s | S > s) \to 0$ and is useful to assess the degrees of dependence at finite levels of s. In the context of bivariate normal, $\bar{c}$ is equal to the correlation coefficient.

In practice, the hypothesis $\bar{c} = 1$ should be tested first. $\bar{c}$ serves then as a measure of dependence if the hypothesis is rejected as in this case the variables are not asymptotically dependent. If the hypothesis can not be rejected, c is computed and serves as the measure for the extremal dependence. In our study, it is of primary interest to determine if hedge funds and traditional assets are asymptotically dependent or if their dependences drop at a certain rate to 0.

Under weak conditions, it can be shown that the estimator for $\bar{c}$ can be expressed as the following:

$$\hat{\bar{c}} = \frac{2}{n_u} \left(\sum_{j=1}^{n_u} \log\left(\frac{z_{(j)}}{u} \right) \right) - 1$$

and $Var(\hat{\bar{c}}) = (\hat{\bar{c}} + 1)^2 / n_u$

where $Z = \min(T, S)$.

$\hat{\bar{c}}$ is the Hill estimator of $\bar{c}$[8]. Moreover, $\hat{\bar{c}}$ is asymptotically normally distributed.

Finally, when the hypothesis $\bar{c} = 1$ cannot be rejected, the measure of dependence c is estimated by:

$$\hat{c} = \frac{u n_u}{n}$$

and $Var(\hat{c}) = \dfrac{u^2 n_u (n - n_u)}{n^3}$.

[8] The Hill (1975) estimator can be easily used in this context as the variables have been transformed to a Fréchet distribution, i.e. the transformed variables exhibit fat tails.

3 Extreme risk in individual asset classes

In this section, we present the data used in our analysis and examine empirically the value at risk and the expected shortfall of stocks, bonds and hedge funds.

3.1 Data

To study the risk of the different asset classes, we use several indices as proxies for these assets: MSCI World for stocks, Citigroup Global Government Index for bonds and the HFRI fund of funds composite index for hedge funds. There are at least four reasons to choose a fund of funds index (FoF) as representative for the hedge fund universe. Firstly, a FoF index is less subject to the different biases in the hedge fund databases such as survivorship bias, backfeeding bias. Secondly, funds of funds invest in funds which are not necessarily listed in any database and thus provide a better and larger coverage of the whole sector. Thirdly, funds of funds are well diversified portfolios. This implies that a FoF index is not sensitive to operational risk. Finally, more and more institutional investors choose funds of funds as vehicles to invest in the hedge fund world.

We consider additional fund of funds indices as the FoF offer has been growing and broadening during the last few years. Hedge Fund Research provides a classification of fund of funds into four different categories. We investigate whether the choice of the FoF index has an impact on the risk behavior of the portfolios built out of stocks, bonds, and hedge funds.

HFRI FOF: Conservative

"FOFs classified as "Conservative" exhibit one or more of the following characteristics: seeks consistent returns by primarily investing in funds that generally engage in more "conservative" strategies such as Equity Market Neutral, Fixed Income Arbitrage, and Convertible Arbitrage"

HFRI FOF: Diversified

"FOFs classified as "Diversified" exhibit one or more of the following characteristics: invests in a variety of strategies among multiple managers"

HFRI FOF: Market Defensive

"FOFs classified as "Market Defensive" exhibit one or more of the following characteristics: invests in funds that generally engage in short-biased strategies such as short selling and managed futures"

HFRI FOF: Strategic

"FOFs classified as "Strategic" exhibit one or more of the following characteristics: seeks superior returns by primarily investing in funds that generally engage in more opportunistic strategies such as Emerging Markets, Sector specific, and Equity Hedge"

As the number of data points is critical in our analysis, we use the maximum period at disposal ranging from January 1990 to August 2003. For each time series, we have 164 monthly returns[9]. This number is rather small compared to some other applications of the EVT to financial market using daily or weekly frequency. One of the main consequences we expect is to obtain rather large confidence intervals for the different estimates.

Table 1 reports the standard statistics for the different indices. The hedge fund indices show the best mean returns and usually smaller volatility than bonds. Moreover, the classification of hedge fund indices is reflected in the different statistics: the Strategic FoF shows the best returns and the highest volatility whereas the Conservative FoF presents the smallest standard deviation. Looking at higher moments, all the fund of funds indices exhibit non normal distributions according to the Jarque Bera statistics. The normality cannot be rejected for the two traditional indices at the usual confidence level (1% and 5%). Significant autocorrelation at lag one is also found in the hedge fund indices (except for the diversified index) as well as for the bond index. At lag two we find evidence for autocorrelation only in the conservative fund of funds index. This is consistent with the fact that market neutral hedge funds present longer autocorrelation (see e.g. Lo (2002)).

[INSERT TABLE 1]

3.2 Risk of the asset classes

We analyse the risk of the different indices by computing their Value-at-Risk and their Expected Shortfall at several confidence levels. We compare the VaR obtained from the EVT with the VaR obtained assuming normality and the modified VaR derived using the Cornish Fisher expansion. The EVT VaR and the EVT ES are computed by estimating the shape parameter of the GDP distribution via maximum likelihood. We first centre the returns by the median of their distribution. In order to avoid data snooping, we define parametrically the threshold u as the product of the percentile p and the empirical standard deviation of the returns. The percentile p is determined by evaluating the mean excess function as suggested by Davison and Smith

[9] This number is rather large compared to what has been previously used in the hedge fund world. For example, Blum, Dacorogna and Jaeger (2003) consider the Tremont indices between January 1994 and december 2002.

(1990). We find that the 81st percentile is a good definition of the threshold for the different analysed time series as it usually gives stable results. On average, we obtain 30 exceedance points (usually ranging between 25 and 40).

Table 2 displays the results for the different methods estimating the Value-at-Risk and the Expected Shortfall. We also report several confidence levels for the VaR and ES estimates as well as the confidence intervals for the different EVT estimates. Several observations are striking. Firstly, the VaR derived assuming normality is always underestimating the one estimated via EVT. The normal VaR is outside the boundaries of the 95% EVT confidence interval 12 times out of 14. The same is true to a lesser extent for the modified VaR. The estimate lies outside the previous boundaries 9 times out of 14. In other words, the normal VaR (or the modified VaR) provides results which are statistically different from the EVT VaR. The modified VaR exhibits also bizarre results for the MSCI World index as its value is always below the one derived assuming normality given the fact that the MSCI World presents negative skewness and slightly excess kurtosis. Secondly, the hedge fund indices (except the Strategic FoF) present lower VaR at 95% confidence level than the bond index. However, when the confidence level is increased to 99%, the picture is reversed except for the Conservative FoF index. This is a direct consequence of the fat tail behaviour of the fund of funds indices. Thirdly, the Expected Shortfall tends to show a more robust behaviour than VaR. Indeed, the ES of the bond index is smaller than the hedge fund ones for all the confidence levels. The only exception is the Conservative FoF index at the 95% confidence level.

[INSERT TABLE 2]

Serial independence and identical distribution (IID) is a key assumption of the extreme value theory. As seen in table one, the hedge fund and bond returns are auto-correlated at lag one[10] (lag two for the Conservative index). As suggested by Kat and Lu (2002) and Okunev and White (2003), we unsmooth the returns using the following formula:

$$R_t = \frac{R_t^* - \hat{r} R_{t-1}^*}{1 - \hat{r}}$$

where R_t is the unsmoothed return, R_t^* is the return computed from the index, and $\hat{r}$ is the coefficient of autocorrelation at lag one.

[10] We also checked for the existence of ARCH effects but did not find any.

After adjusting the returns according to the first autocorrelation, we check for additional autocorrelation at longer horizons and do not find any evidence even in the case of the conservative FoF index. As a consequence, we do not proceed to further adjustments. Table 3 reports the results of the Value-at-Risk and the Expected Shortfall computed on the unsmoothed time series. As expected, VaR and ES are increased for the auto-correlated indices. The increase of VaR is not only due to an increase in the underlying variance but also to modification of the shape parameter. The 95% confidence intervals of the different estimates are wider than the ones computed with raw returns. Finally, the order of VaR and ES is not modified: the difference between the VaR of the MSCI index and the others indices are still significantly reduced.

[INSERT TABLE 3]

4 Extreme risk in portfolios of hedge funds, stocks and bonds

4.1 Portfolio risk

In order to analyse how hedge funds, stocks, and bonds fit together, we start by building portfolios out of the different asset classes. Contrary to Amin and Kat (2003), we do not run optimisation for at least three reasons. Firstly, any optimisation framework relies on the definition of expected returns, which are particularly prone to errors. As a consequence, the choice of expected returns coming from a model or from a historical perspective influences the optimal weights of hedge funds, stocks, and bonds in the portfolio. Secondly, optimisation methods are very sensitive to errors in the different estimates and tend to exacerbate the impact of the errors on the optimal weights (see e.g. Michaud (1998) in the traditional context of mean-variance). Finally, the behaviour of institutional investors is not well captured by an optimisation framework. Indeed, institutional investors tend to favour limited investment (between 1% and 5%) when considering the inclusion of a new asset class in their portfolio.

We build different sets of portfolios by choosing the initial composition between stocks and bonds. Eleven sets are defined, where the allocation to stocks (bonds) is ranging from 0% (100%) to 100% (0%) with a step of 10%. In each of the sets, we add different levels of hedge funds (0%, 1%, 5%, 10%, 15%, 20% up to 100% with a step of 5%). When hedge funds are added to the portfolio, the proportion of stocks (or bonds) is kept constant in the traditional part of the portfolio. For example, if a set is built with 20% stocks and 80% bonds, adding 20% hedge funds will decrease the weight of stocks to 20% * 80%= 16% and the weight of bonds to 80% *80%=0.64. In total, we analyse 242 portfolios for a given fund of funds index which corresponds to 1210 portfolios for the five fund of funds indices. This method provides more information than standard optimisation frameworks.

Figure 1a reports the results (95% VaR and 95% ES) obtained for the Composite FoF. Two types of returns are considered for this comparison: raw returns and unsmoothed returns. The unsmoothed return series of the different portfolios are constructed using the individual unsmoothed return series of each index. In this context, each portfolio is constructed using the same underlying returns. However, this implies that we are not considering any possible cross-auto-correlation between the return series which might induce auto-correlation at the portfolio level. This is why we have also checked the linear autocorrelation for each portfolio: no significant autocorrelation has been found at the portfolio level[11]. As shown in Figure 1a, the form and behaviour of VaR and ES are not radically affected by the type of returns. In Figures 1b and 1c, we only report results based on unsmoothed returns.

The analysis of Figures 1a, 1b and 1c clearly shows three different parts, except for the market defensive fund of funds. Firstly, when the traditional portfolios contain mostly stocks, VaR and ES are strictly decreasing to the VaR and the ES of the individual FoF. In other words, these portfolios should contain as much hedge fund as possible. Secondly, when the traditional portfolios have around 40% stocks and 60% bonds, VaR and ES do not change when more hedge funds are added to the portfolio. In other words, the addition of hedge funds does not alter the risk characteristics of the portfolio. Including hedge funds in the portfolio may still be worth due to return enhancement. Finally, when the traditional portfolio contains mostly bonds, diversification effects can be achieved. We find that an optimal composition between 50% and 60% in hedge funds and between 40% and 50% in bonds for the overall portfolio. Moreover, the reduction of VaR and ES is significant at a 5% confidence level. For example, the VaR (ES) of a bond only portfolio is 3.79% (4.57%). When 50% Composite fund of funds are added to this portfolio, VaR (ES) drops to 2.48% (3.39%) and the upper limit of the confidence interval is 3.01% for VaR (4.39% for ES).

Figure 1c reports a different behaviour when market defensive fund of funds are used. In the case of the addition of market defensive fund of funds to the traditional portfolio, we find an optimal composition (optimal level of hedge funds) for each traditional portfolio. The level of hedge funds to be added is function of the composition of the traditional benchmark. In other words, the market defensive FoF category provides a different risk profile presenting

[11] The results are not reported in the paper but are available upon request.

diversification effects whatever the initial traditional portfolio. The market defensive FoFs are overweighed toward managed futures and short sellers. Our findings are consistent with Kat (2002) claiming that managed futures reduce substantially the risk of traditional portfolios. From a risk perspective, a hedge fund portfolio should contain managed futures in order to diversify the extreme risk in the traditional portfolio part.

As the choice of the threshold is critical in the EVT framework, we explore the impact of this choice. As already mentioned, we use a parametric definition of the threshold corresponding to the product of the percentile p of the standard normal distribution and the empirical standard deviation of the return time series. We consider three different percentiles: 75^{th}, 81^{st} (the usual one), and 85^{th}. Figure 2 summarises the results for some portfolios[12].We do not find significant differences between the results of the various thresholds.

4.2 Extreme dependences

So far, we have been showing that hedge funds fit well in a traditional portfolio. Indeed, they are able to reduce the risk of the different traditional portfolios. However, when building portfolios, the dependence between hedge funds and the other asset classes is treated as endogenous. In this section, we explicitly evaluate the extreme dependence between the different assets as defined in the context of multivariate EVT.

We start by estimating the parameter $\bar{c}$ for each pair of assets without boundaries. We perform the following test:

H_N: $\bar{c} = 1$

H_A: $\bar{c} < 1$

If the null hypothesis cannot be rejected the parameter $\bar{c}$ is not describing correctly the extremal dependence. In this case, we estimate the tail dependence coefficient c .

[INSERT TABLE 4a, 4b, 4c]

Tables 4a, 4b and 4c report the different values for the extreme dependence coefficients and their associated standard errors. The parameters $\bar{c}$ displayed in table 4a are of the same

[12] The results for all the portfolios are the same and are available upon request.

magnitude or even smaller than the ones reported by Poon, Rockinger and Tawn (2003) on different stock markets. In particular, none of the parameter is above 1 even though they were not constrained. However, the standard errors in our study are larger as our sample size is quite small. Despite this fact, several conclusions can be drawn. Firstly, we do not find any evidence of asymptotic dependence between hedge funds and bonds. Market Defensive FoF index is even negatively related to the Bond index. These findings may be related to the reduction of VaR and ES in portfolios dominated by bonds and hedge funds. The optimal composition found previously is a direct consequence of the absence of extreme dependence between hedge funds and bonds.

Secondly, the stock market index and some hedge funds indices, namely the Composite, Conservative, and Strategic FoF indices, are asymptotically related. The Diversified and Market Defensive do not exhibit extreme dependence with the stock market. In the context of the Diversified index, the constituents of the index are well diversified across the strategies covered by hedge fund. This implies that this type of FoF is exposed to a wide range of sources of risk which reduces the link to the stock market. The results obtained with the Market Defensive index are consistent with previous findings showing that Managed Futures provide some downside protection to equity markets (see e.g. Kat (2002)). Again, this is consistent with the strong reduction of VaR and ES when Market Defensive FoF is added to portfolios containing mainly equities. Moreover, the Market Defensive index exhibits a different behavior from the other FoF indices. Indeed, all the hedge fund indices, except the Market Defensive, present one to one asymptotic tail dependence.

Table 4b displays the estimated asymptotic dependence coefficient c for the pairs displaying a parameter $\bar{c}$ not statistically different from 1. The links between hedge funds indices are stronger and bigger than between hedge fund indices and the stock market index. In table 4c, we try to determine the cause of the dependence in the tails between the different indices. We eliminate the observation for August 1998 which corresponds to the Russian crisis. This crisis is mainly related to a global liquidity crisis affecting primarily the relative value arbitrage hedge funds like LTCM as well as the equity markets. All the tail dependence coefficients $\bar{c}$ are reduced and the extreme dependence between hedge fund indices and stock market is not significant anymore. However, most of the hedge fund indices are remaining dependent in the extreme. We conjecture that hedge funds are able to control their market risk but may be strongly impacted by extremely bad liquidity conditions.

Conclusion

In this paper we have been analysing the behaviour in the extreme left tail of funds of hedge funds, stock and bonds indices. Contrary to Amin and Kat (2003), we do not analyse individually the first three moments of the distribution without measuring substitution effects. Instead we use the Extreme Value Theory in order to infer the Value-at-Risk and the Expected Shortfall. This enables us to evaluate and to compare the risk of the different asset classes taking higher moments into account.

We find that the benefits of the inclusion of hedge funds in a traditional portfolio depend on the initial composition of the portfolio and on the type of the added hedge funds. When the initial portfolio is dominated by stocks, every addition of hedge fund is reducing the risk as measured by VaR or ES. If the added funds of hedge funds are biased toward managed futures, the risk reduction of an equity dominated portfolio is bigger and an optimal level minimising the risk is found with around 80% invested in the fund of hedge funds. When the initial portfolio is composed mostly of bonds, we find also an optimal composition minimising the risk with around 50% invested in hedge funds.

We then examine the asymptotic dependence between hedge funds and traditional investment using two non-parametric measures. We do not find any statistical evidence of dependence between hedge funds and bonds. Some fund of funds indices and stock markets present statistically significant asymptotic dependence. This is not the case for fund of hedge funds which are biased toward managed futures. This finding is consistent with the diversification effects documented when mixing equities and Market Defensive Funds of Funds. Furthermore we document that asymptotic dependence between hedge funds and stocks is the consequence of the August 1998 event which is strongly related to liquidity crisis. As a consequence it would be very interesting to test if liquidity is responsible for the extreme link between equity indices and hedge funds. This is left for future research.

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Table 1: Standard statistics for the different indices

	Stocks	Bonds	Composite	Conservative	Diversified	Defensive	Strategic
Mean	0.55%	0.62%	0.82%	0.72%	0.76%	0.84%	1.10%
Volatility	4.33%	1.86%	1.68%	0.97%	1.80%	1.75%	2.72%
Skewness	-0.40	0.26	-0.27	-0.52	-0.10	0.19	-0.39
Excess Kurtosis	0.27	0.10	4.06	3.57	3.96	1.35	3.30
JB Stat	4.82	1.93	**114.73**	**94.35**	**107.50**	**13.37**	**78.59**
Min	-13.32%	-3.63%	-7.47%	-3.88%	-7.75%	-5.42%	-12.11%
Max	10.55%	5.94%	6.85%	3.96%	7.73%	7.38%	9.47%
Correlation lag 1	-0.01	**0.24**	**0.31**	**0.31**	**0.32**	0.10	**0.29**
Correlation lag 2	-0.07	-0.02	0.12	**0.21**	0.10	0.01	0.10
Correlation lag 3	0.01	0.01	0.01	0.05	-0.03	0.05	0.05

The table reports the standard statistics for the monthly returns of the indices used in the study between January 1990 and August 2003. JB Stat designates the Jarque Bera statistics which follows a chi-square distribution with two degrees of freedom. Values in bold indicate a 1% significance.

Stocks are proxied by the MSCI World index, Bonds by the Citigroup Global Government Bond index, Composite by the HFRI Fund of Funds Composite index, Conservative by the HFRI FOF: Conservative index, Diversified by the HFRI FOF: Diversified index, Defensive by the HFRI FOF: Market Defensive index, and Strategic by the HFRI FOF: Strategic index.

Table 2: Value at Risk and Expected Shortfall for the raw returns

	Stocks	Bonds	Composite	Conservative	Diversified	Defensive	Strategic
Shape x	-0.18	-0.58	0.16	-0.03	0.21	0.06	0.12
	Confidence level : 95%						
VaR EVT	8.04	3.00	2.46	1.73	2.56	2.50	4.38
VaR EVT Conf.	[7.11 ;9.52]	[2.90 ;3.32]	[2.09 ;3.11]	[1.46 ;2.18]	[2.16 ;3.27]	[2.16 ;3.07]	[3.74 ;5.43]
VaR Normal	6.58	2.43	1.94	0.87	2.20	2.04	3.37
VaR Modified	6.06	2.56	1.67	0.65	2.01	2.08	2.88
ES EVT	10.24	3.51	3.74	2.49	4.02	3.51	6.32
ES EVT Conf.	[8.84 ;12.47]	[3.37 ;3.94]	[2.91 ;5.17]	[1.99 ;3.31]	[3.08 ;5.67]	[2.85 ;4.60]	[5.05 ;8.40]
	Confidence level : 99%						
VaR EVT	11.67	3.85	4.42	2.96	4.76	4.10	7.41
VaR EVT Conf.	[9.97 ;14.38]	[3.68 ;4.35]	[3.36 ;6.29]	[2.33 ;4.01]	[3.55 ;6.89]	[3.25 ;5.50]	[5.79 ;10.06]
VaR Normal	9.54	3.70	3.08	1.52	3.43	3.23	5.23
VaR Modified	8.28	4.05	4.30	1.86	4.97	4.00	6.39
ES EVT	13.32	4.05	6.08	3.68	6.80	5.20	9.78
ES EVT Conf.	[11.27 ;16.58]	[3.87 ;4.60]	[4.42 ;8.96]	[2.84 ;5.10]	[4.83 ;10.25]	[4.01 ;7.18]	[7.40 ;13.68]

The table reports the Value-at-Risk and the Expected Shortfall for the monthly returns of the indices used in the study between January 1990 and August 2003. All the numbers are expressed in percent except the shape parameter. The reported confidence intervals for the EVT estimate are fixed at 95%.
Stocks are proxied by the MSCI World index, Bonds by the Citigroup Global Government Bond index, Composite by the HFRI Fund of Funds Composite index, Conservative by the HFRI FOF: Conservative index, Diversified by the HFRI FOF: Diversified index, Defensive by the HFRI FOF: Market Defensive index, and Strategic by the HFRI FOF: Strategic index.

Table 3: Value at Risk and Expected Shortfall for the unsmoothed returns

	Stocks	Bonds	Composite	Conservative	Diversified	Defensive	Strategic
Shape x	-0.20	-0.67	0.26	0.08	0.31	-0.01	0.18
	Confidence level : 95%						
VaR EVT	7.98	3.82	3.09	2.32	3.25	4.09	5.37
VaR EVT Conf.	[7.08 ;9.45]	[3.75 ;4.21]	[2.65 ;3.87]	[1.95 ;2.92]	[2.78 ;4.14]	[3.54 ;5.01]	[4.60 ;6.66]
VaR Normal	6.67	3.27	2.97	1.47	3.37	3.40	4.92
VaR Modified	6.14	3.38	2.59	1.14	3.06	3.23	4.35
ES EVT	10.15	4.60	4.79	3.37	5.36	5.62	7.89
ES EVT Conf.	[8.80 ;12.34]	[4.50 ;5.16]	[3.70 ;6.73]	[2.68 ;4.50]	[4.01 ;7.90]	[4.62 ;7.27]	[6.26 ;10.60]
	Confidence level : 99%						
VaR EVT	11.56	5.11	5.61	3.99	6.56	6.28	9.23
VaR EVT Conf.	[9.92 ;14.22]	[4.98 ;5.77]	[4.21 ;8.11]	[3.11 ;5.42]	[5.29 ;8.66]	[4.55 ;9.55]	[7.15 ;12.70]
VaR Normal	9.59	4.88	4.54	2.37	5.08	5.15	7.42
VaR Modified	8.33	5.23	6.02	2.77	6.87	5.36	8.97
ES EVT	13.13	5.38	8.19	5.19	8.06	9.77	12.59
ES EVT Conf.	[11.17 ;16.32]	[5.25 ;6.10]	[5.80 ;12.45]	[3.94 ;7.22]	[6.35 ;10.87]	[6.58 ;15.76]	[9.37 ;17.97]

The table reports the Value-at-Risk and the Expected Shortfall for the monthly returns of the indices used in the study between January 1990 and August 2003. All the numbers are expressed in percent except the shape parameter. The reported confidence intervals for the EVT estimate are fixed at 95%.
Stocks are proxied by the MSCI World index, Bonds by the Citigroup Global Government Bond index, Composite by the HFRI Fund of Funds Composite index, Conservative by the HFRI FOF: Conservative index, Diversified by the HFRI FOF: Diversified index, Defensive by the HFRI FOF: Market Defensive index, and Strategic by the HFRI FOF: Strategic index.

Table 4a: Estimates of the $\overline{c}$ parameters

	Stocks	Bonds	Composite	Conservative	Diversified	Defensive	Strategic
Stocks		**0.19** **(0.23)**	0.53 (0.31)	0.51 (0.30)	**0.46** **(0.29)**	**0.18** **(0.24)**	0.68 (0.34)
Bonds			**0.12** **(0.22)**	**0.22** **(0.25)**	**0.24** **(0.25)**	**-0.10** **(0.18)**	**0.29** **(0.26)**
Composite				0.80 (0.36)	0.73 (0.35)	**0.41** **(0.28)**	0.70 (0.34)
Conservative					0.90 (0.38)	**0.35** **(0.27)**	0.67 (0.33)
Diversified						**0.36** **(0.27)**	0.77 (0.35)
Defensive							**0.25** **(0.25)**
Strategic							

The table reports the estimate and the standard error (in parenthesis) of the parameter $\overline{c}$ between January 1990 and August 2003. Bold value indicates that the parameter $\overline{c}$ is statistically smaller than 1 using a one-sided test with confidence level of 5%.

Stocks are proxied by the MSCI World index, Bonds by the Citigroup Global Government Bond index, Composite by the HFRI Fund of Funds Composite index, Conservative by the HFRI FOF: Conservative index, Diversified by the HFRI FOF: Diversified index, Defensive by the HFRI FOF: Market Defensive index, and Strategic by the HFRI FOF: Strategic index.

Table 4b: Estimates of the c parameters

	Stocks	Bonds	Composite	Conservative	Diversified	Defensive	Strategic
Stocks			0.45 (0.08)	0.45 (0.08)			0.43 (0.08)
Bonds							
Composite				0.58 (0.11)	0.82 (0.15)		0.71 (0.13)
Conservative					0.49 (0.09)		0.50 (0.09)
Diversified							0.64 (0.12)
Defensive							
Strategic							

The table reports the estimate and the standard error (in parenthesis) of the parameter c between January 1990 and August 2003. The value of the parameter c is only reported if the corresponding parameter $\overline{c}$ was not statistically strictly smaller than 1.

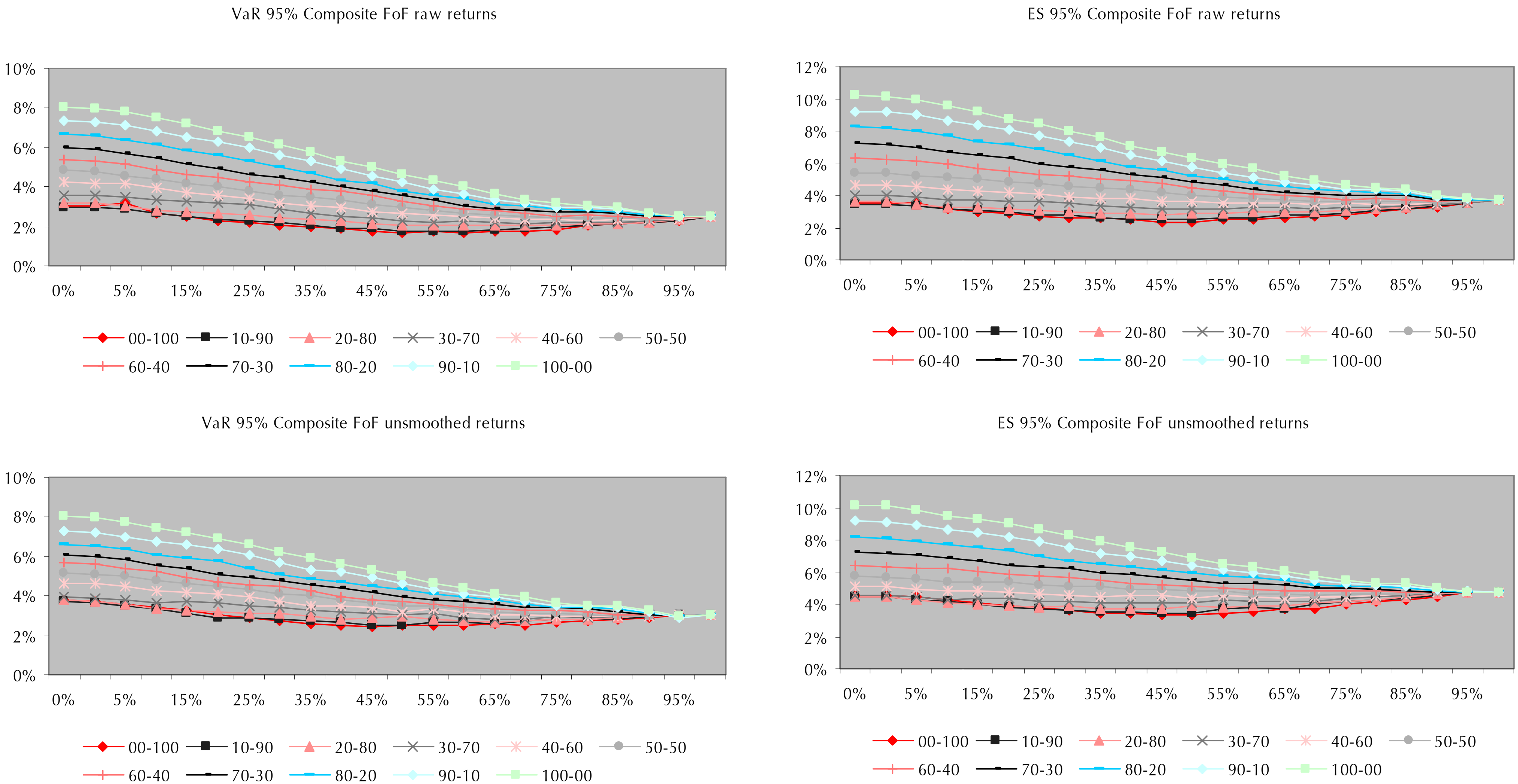
Stocks are proxied by the MSCI World index, Bonds by the Citigroup Global Government Bond index, Composite by the HFRI Fund of Funds Composite index, Conservative by the HFRI FOF: Conservative index, Diversified by the HFRI FOF: Diversified index, Defensive by the HFRI FOF: Market Defensive index, and Strategic by the HFRI FOF: Strategic index.

Table 4c: Estimates of the $\overline{c}$ parameters when August 1998 is removed

	Stocks	Bonds	Composite	Conservative	Diversified	Defensive	Strategic
Stocks		**0.20** **(0.24)**	**0.31** **(26)**	**0.45** **(0.29)**	**0.29** **(0.26)**	**-0.15** **(0.17)**	**0.45** **(0.29)**
Bonds			**0.18** **(0.24)**	**0.28** **(0.26)**	**0.30** **(0.26)**	**-0.11** **(0.18)**	**0.38** **(0.29)**
Composite				0.66 (0.33)	0.69 (0.34)	**0.22** **(0.24)**	0.55 (0.31)
Conservative					0.72 (0.34)	**0.17** **(0.23)**	**0.42** **(0.29)**
Diversified						**0.15** **(0.23)**	0.74 (0.35)
Defensive							**0.03** **(0.21)**
Strategic							

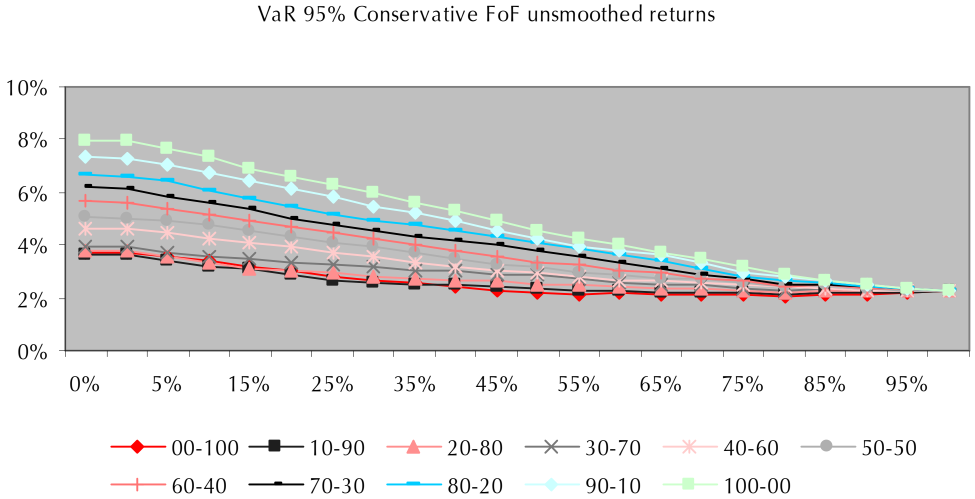
The table reports the estimate and the standard error (in parenthesis) of the parameter $\overline{c}$ between January 1990 and August 2003. August 1998 has been removed from the sample. Bold value indicates that the parameter $\overline{c}$ is statistically smaller than 1 using a one-sided test with confidence level of 5%. Stocks are proxied by the MSCI World index, Bonds by the Citigroup Global Government Bond index, Composite by the HFRI Fund of Funds Composite index, Conservative by the HFRI FOF: Conservative index, Diversified by the HFRI FOF: Diversified index, Defensive by the HFRI FOF: Market Defensive index, and Strategic by the HFRI FOF: Strategic index.

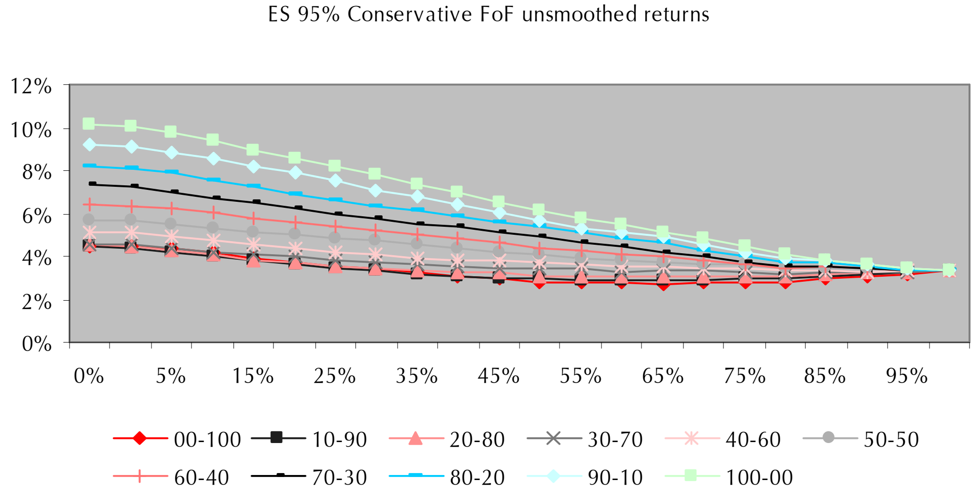
Figure 1a: Evolution of the Value-at-Risk and the Expected Shortfall with respect to the investment in hedge funds

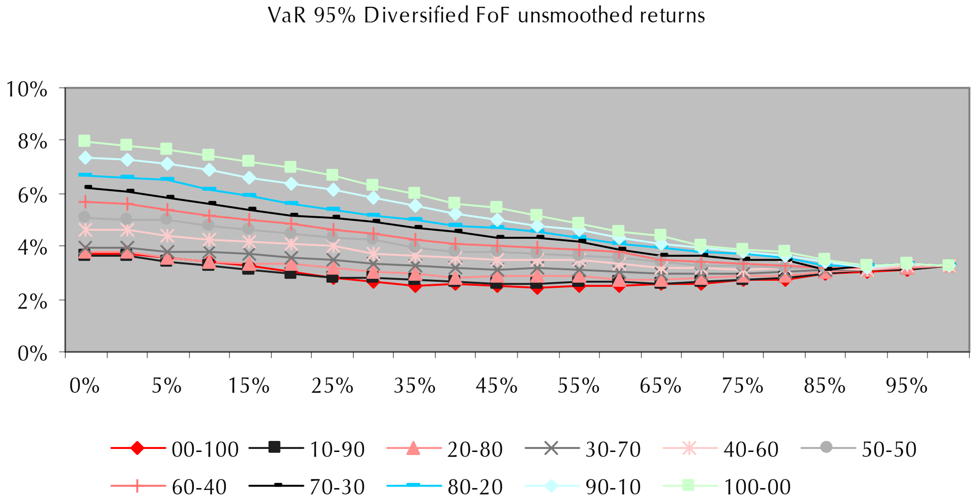


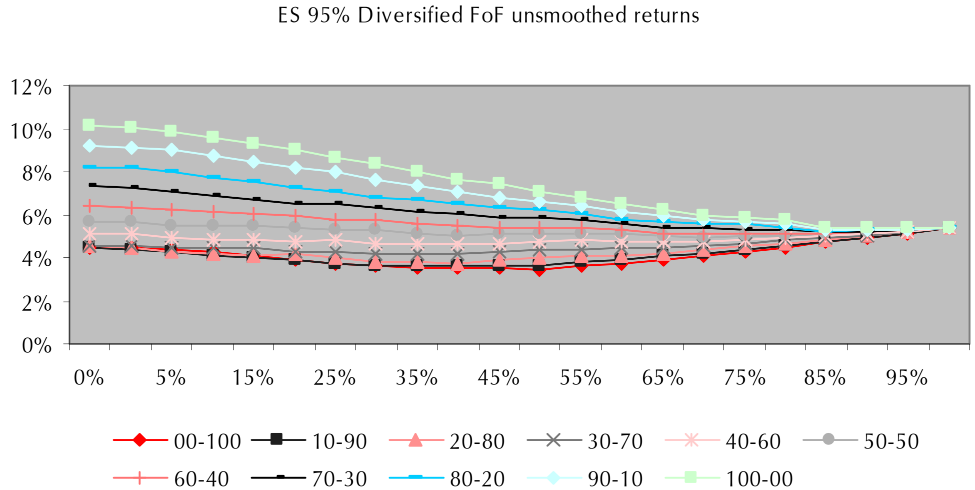
Each line represents a particular set of portfolio (weights of stocks, weights of bonds). The x-line depicts the weights of hedge funds.

Figure 1b: Evolution of the Value-at-Risk and the Expected Shortfall with respect to the investment in hedge funds



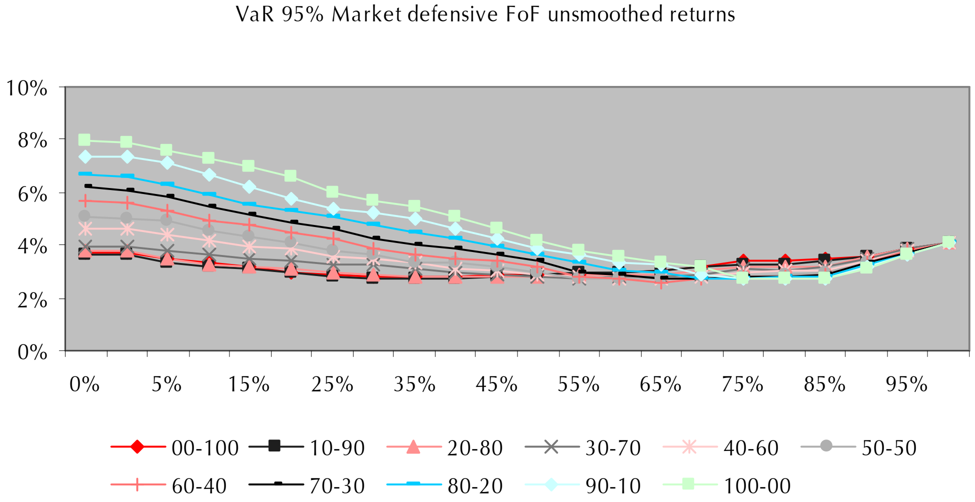


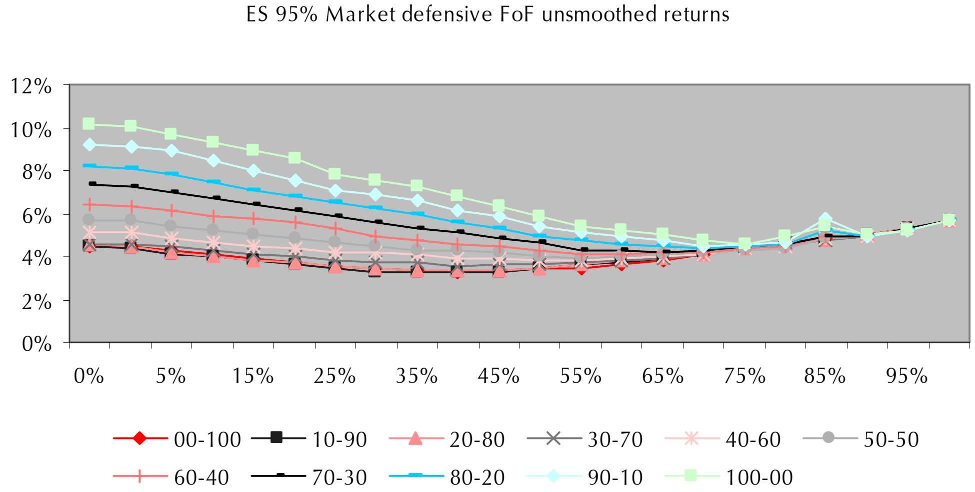


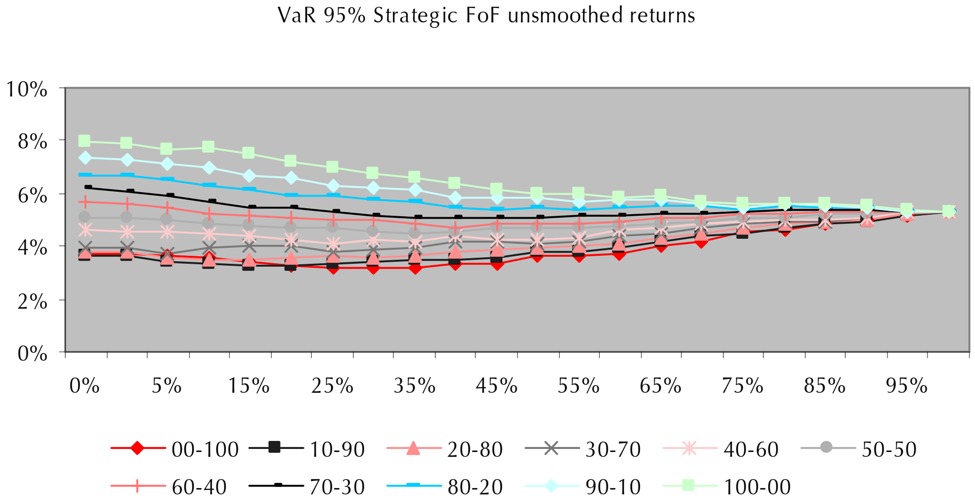


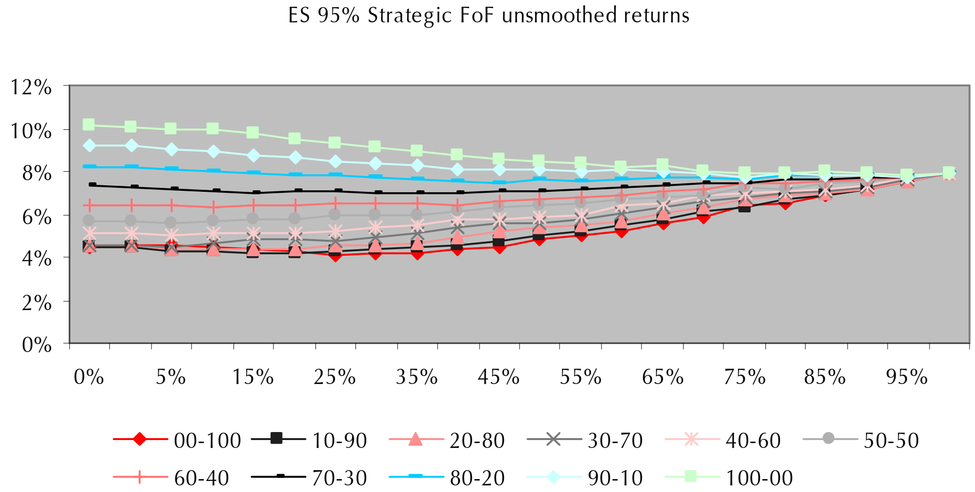
Each line represents a particular set of portfolio (weights of stocks, weights of bonds). The x-line depicts the weights of hedge funds.

Figure 1c: Evolution of the Value-at-Risk and the Expected Shortfall with respect to the investment in hedge funds



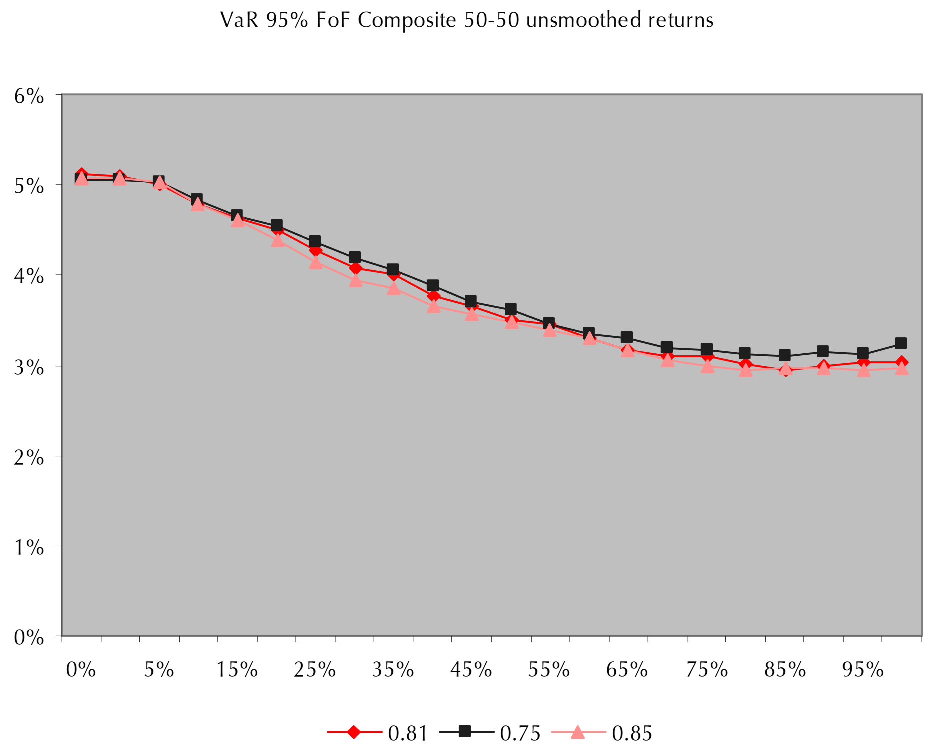


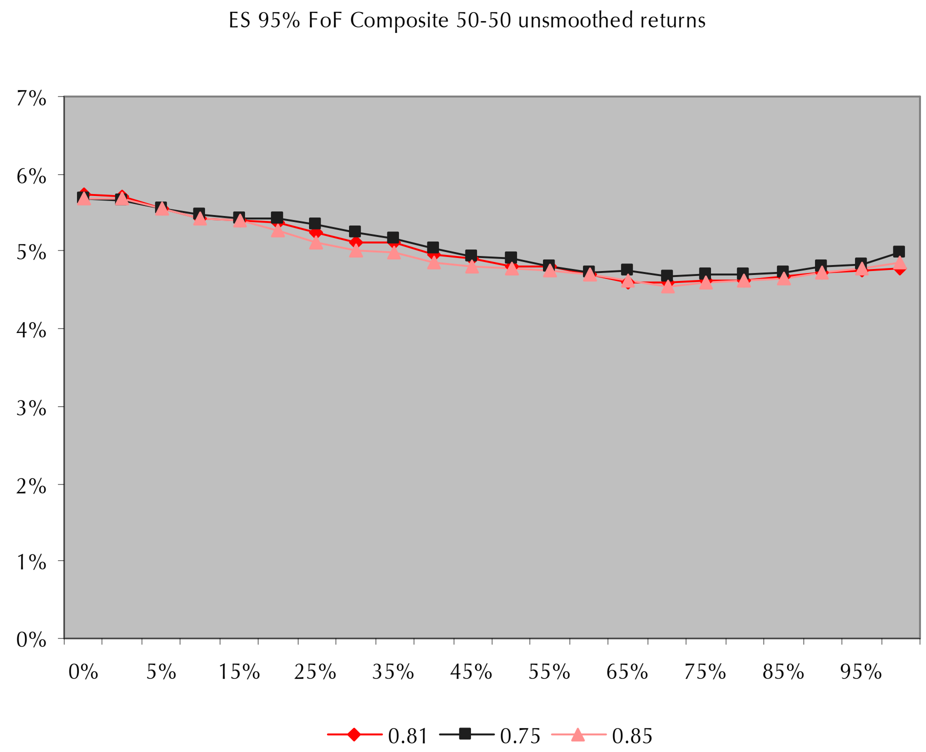




Each line represents a particular set of portfolio (weights of stocks, weights of bonds). The x-line depicts the weights of hedge funds.

Figure 2: Sensitivity to the choice of the threshold





The figure displays the change in the Value-at-Risk and in the Expected Shortfall according to the choice of the threshold for a traditional portfolio (50% bonds, 50% stocks). The x-line depicts the weights of hedge funds. The values reported as the bottom of the graphs corresponds to a percentile (81st, 75th, 85th). We define the threshold as the x-th percentile of a normal distribution whose variance is given by the empirical distribution of the returns.