Asset Performance Evaluation with the Mean-Variance Ratio

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Abstract:

Asset performance evaluation is one of the most important areas in investment analysis. In order to compare the performance among assets, several statistics have been developed; and among them, the Sharpe-ratio statistic is the most prevalent. However, the major limitation of the Sharpe-ratio statistic is that its distribution is only valid asymptotically, but not valid for small samples. Nevertheless, it is important in finance to test the performance among assets for small samples. To further serve this purpose, we develop both one-sided and two-sided mean-variance-ratio statistics to evaluate the performance among the assets for small samples. In this paper we further prove that our proposed statistics provide uniformly most powerful unbiased tests. We illustrate the superiority of our proposed test over the traditional Sharpe-ratio test by applying both tests to analyze the performance of funds from Commodity Trading Advisors. Our findings show that while the traditional Sharpe-ratio test concludes most of the CTA funds being analyzed as being indistinguishable in their performance, our proposed statistics show that some funds outperform the others. On the other hand, when we apply the Sharpe-ratio statistic on some other funds, we find that the statistic indicates that one fund is significantly outperforming another fund even though the difference between the two funds is insignificantly different from zero and/or even changes directions. However, when our proposed mean-variance-ratio statistic is applied, we could detect the change in the difference. This shows the superiority of our proposed statistic in revealing short term performance and in return, enables investors to make better decisions about their investments.

Keywords: Sharpe ratio, performance hypothesis testing, Normal distribution, uniformly most powerful unbiased test, CTA funds.

1 Introduction

The pioneer work of Markowitz (1952, 1959) on the mean-variance portfolio optimisation procedure has been widely used in both Economics and Finance to analyze how people make their choices concerning risky investments. The Markowitz efficient frontier also provides the basis for many important financial economics advances, including the Sharpe-Linter Capital Asset Pricing Model (Sharpe, 1964; Lintner, 1965) and the well-known optimal one-fund theorem (Tobin, 1958). Originally motivated by the mean-variance analysis, the optimal one-fund theorem and the Sharpe-Linter Capital Asset Pricing Model, the Sharpe ratio, the ratio of the excess expected return to its volatility or standard deviation, is one of the most commonly used statistics in the mean-variance framework. The Sharpe ratio is now widely used in many different areas in Finance and Economics, from the evaluation of portfolio performance to market efficiency test (see, for example, Levy, 1972; Cumby and Glen, 1990; Grinblatt and Titman, 1994; Ofek and Richardson, 2003; Agarwal and Naik, 2004).

Although the Sharpe ratio has been widely used with a myriad of interpretations, only a few literary papers study its statistical properties. Jobson and Korkie (1981) first develop a Sharpe-ratio statistic to test for the equality of two Sharpe ratios, whereby the statistic is being further modified and improved by Cadsby (1986) and Memmel (2003). On the other hand, by invoking the standard econometric methods with several different sets of assumptions imposing on the statistical behavior of the return series, Lo (2002) derives the asymptotic statistical distribution for the Sharpe-ratio estimator. With this statistical distribution, he shows that confidence intervals, standard errors, and hypothesis tests can be computed for the estimated Sharpe ratios in much the same way as regression coefficients such as portfolio alphas and betas are computed.

The Sharpe-ratio test statistics developed by Jobson and Korkie (1981), Cadsby (1986), Lo (2002) and Memmel (2003) are important as they provide a formal statistical comparison for the performances among portfolios. However, as the Sharpe-ratio statistic possesses only the asymptotic distribution, one could only obtain its properties for large samples, but not for small samples. Nevertheless, it is important in finance to compare the performance of assets by using small samples, especially before and after markets change their directions, in which only small samples could be used to predict the assets' future performance. Also it is, sometimes, not so meaningful to measure Sharpe ratios for too long periods as the means and standard deviations of the underlying assets could be empirically non-stationary. The main obstacle in developing the Sharpe-ratio test for small samples is that it is impossible to obtain a uniformly most powerful unbiased (UMPU) test to check for the equality of Sharpe ratios in case of small samples. To circumvent this problem, we propose, in this paper, to use the mean-variance ratio for the comparison. With this suggestion, we fill in the gaps among published literary works that discuss the evaluation of the performance of assets for small samples by invoking both one-sided and two-sided UMPU mean-variance-ratio tests.

To demonstrate the superiority of our proposed test over the traditional Sharpe-ratio test, we apply both tests to analyze the performance of funds from Commodity Trading Advisors (CTAs) which involve the trading of commodity futures, financial futures and options on futures (Brorsen and Irwin, 1985; Elton et al., 1987; Kat, 2004). There are many studies analyzing CTAs, in which some (see, for example, Elton et al., 1987) conclude that CTAs offer neither an attractive alternative to bonds and stocks nor a profitable addition to a portfolio of bond and stocks. Whereas, others, (see, for example, Brorsen and Irwin, 1985) conclude that commodity funds produce favorable and appropriate investment returns. We choose analyzing CTAs to illustrate the theories that we are putting forth in this paper simply because CTAs have become very popular with many investors, including universities; the number of universities increasingly allotting their university endowment funds to CTAs has grown significantly (Kat, 2004).

Applying the traditional Sharpe-ratio test, we fail to reject the possibility of having any significant difference among most of the CTA funds; thereby implying that most of the CTA funds being analyzed are indistinguishable in their performance. This conclusion may not necessarily be accurate as the insensitivity of the Sharpe-ratio test is well known due to its limitation on the analysis for small samples. Thus, we invoke our proposed statistic, which is valid for small samples as well as large samples, to the analysis; the conclusion drawn from our proposed test will then be meaningful. As expected, contrary to the conclusion drawn by applying Sharpe-ratio test, our proposed mean-variance-ratio test shows that the mean-variance ratios of some CTA funds are different from the others. This means that some CTA funds outperform other CTA funds in the market. Thus, the test developed in our paper provides more meaningful information in the evaluation of the portfolios' performance and enable investors to make wiser decisions in their investments.

On the other hand, when we apply the Sharpe-ratio statistic to some other funds, we find that the statistic indicates that one fund significantly outperforms another fund even though the difference between the two funds becomes insignificantly small or even changes direction. This shows that the Sharpe-ratio statistic may not be able to reveal the real short run performance of the funds. On the other hand, in our analysis, we find that our proposed mean-variance-ratio statistic could reveal such changes. This shows the superiority of our proposed statistic in detecting short term performance, and in return, enabling the investors to make better decisions in their various investments.

The rest of the paper is organized as follows: Section 2 develops the theory for both one-sided and two-sided mean-variance-ratio tests and studies their properties. In Section 3, we demonstrate the superiority of our proposed tests over the traditional Sharpe-ratio tests by applying both tests to analyze the CTAs. This is then followed up by Section 4 which summarizes our conclusions and shares our insights.

2 The Theory

Let X_i and Y_i $(i = 1, 2, \cdots, n)$ be independent excess returns drawn from the corresponding normal distributions $N(\mu, \sigma^2)$ and $N(\eta, \tau^2)$ with joint density $p(x, y)$ such that

$$p(x,y) = k \times \exp\left(\frac{\mu}{\sigma^2}\sum x_i - \frac{1}{2\sigma^2}\sum x_i^2 + \frac{\eta}{\tau^2}\sum y_i - \frac{1}{2\tau^2}\sum y_i^2\right) \tag{1}$$

where $k = (2\pi\sigma^2)^{-n/2}(2\pi\tau^2)^{-n/2}\exp(-\frac{n\mu^2}{2\sigma^2})\exp(-\frac{n\eta^2}{2\tau^2})$.

To evaluate the performance of the prospects X and Y, financial practitioners and academicians are interested in testing the hypotheses

$$H_0^* : \frac{\mu}{\sigma} \leq \frac{\eta}{\tau} \quad \text{versus} \quad H_1^* : \frac{\mu}{\sigma} > \frac{\eta}{\tau} \,. \tag{2}$$

to compare the performance of their corresponding Sharpe ratios, $\frac{\mu}{\sigma}$ and $\frac{\eta}{\tau}$, the ratios of the excess expected returns to their standard deviations.

Rejecting H_0^* implies X to be the better investment prospect with larger Sharpe ratio that X has either larger excess mean return or smaller standard deviation or both. Jobson and Korkie (1981) and Memmel (2003) develop test statistics to test the hypotheses in (2) for large samples but their tests are not appropriate for testing small samples as the distribution of their test statistics is only valid asymptotically, but is not valid for small samples. However, it will be important in finance to test the hypotheses in (2) for small samples to provide useful investment information to investors. Furthermore, as it is impossible to obtain any other UMPU test statistics to test the inequality of the Sharpe ratios in (2) for small samples, in this paper we propose to alter the hypothesis to test the inequality of the mean-variance ratios as shown in the following:

$$H_0 : \frac{\mu}{\sigma^2} \leq \frac{\eta}{\tau^2} \quad \text{versus} \quad H_{11} : \frac{\mu}{\sigma^2} > \frac{\eta}{\tau^2} \,. \tag{3}$$

We will develop the UMPU test statistic to test the above hypotheses in the paper. Rejecting H_0 suggests X to be the better investment prospect as X possesses either smaller variance or bigger excess mean return or both. As, sometimes, investors do conduct the two-sided test to compare the mean-variance ratios, to complete the theory, we also consider the following hypotheses in this paper:

$$H_0 : \frac{\mu}{\sigma^2} = \frac{\eta}{\tau^2} \quad \text{versus} \quad H_{12} : \frac{\mu}{\sigma^2} \neq \frac{\eta}{\tau^2} \,. \tag{4}$$

Remark 1 *One may think that the mean-variance ratio could be less favorable than* *the Sharpe ratio as the former is not scale invariant while the latter is. However, in some financial processes, the mean change of the process in a short period of time is proportional to its variance change. For example, many financial processes could be characterized by the diffusion process such that the process of stock prices formulated as*

$$dY_t = \mu^P(Y_t)dt + \sigma(Y_t)dW_t^P,$$

(see, Cheridito et al., 2003), where μ^P *is an* N *dimensional function,* σ *is an* $N \times N$ *matrix function and* W_t^P *is an* N*-dimensional standard Brownian motion under the objective probability measure* P*. Under this model, the conditional mean of the increment* dY_t *given* Y_t *is* $\mu^P(Y_t)dt$ *and the covariance matrix is* $\sigma(Y_t)\sigma^T(Y_t)dt$*. When* $N = 1$*, the Sharpe ratio will be close to 0 while the mean-variance ratio will be independent of* dt*. Thus, when the time period dt is small, it is better to consider the mean-variance ratio rather than the Sharpe ratio.*

In this paper, we develop both one-sided UMPU test and two-sided UMPU test to check the equality of the mean-variance ratios for comparing the performances of different prospects with hypotheses stated in (3) and (4) respectively. We first state the one-sided UMPU test for the mean-variance ratios as follows:

Theorem 1 *Let* X_i *and* Y_i $(i = 1, 2, \cdots, n)$ *be independent random variables with joint distribution function defined in (1). For the hypotheses setup in (3), there exists a UMPU level-*α *test with the critical function* $\phi(u, t)$ *such that*

$$\phi(u,t) = \begin{cases} 1, & \text{when } u \geq C_0(t) \\ 0, & \text{when } u < C_0(t) \end{cases} \tag{5}$$

where C_0 *is determined by*

$$\int_{C_0}^{\infty} f_{n,t}^*(u)\ du = K_1\,; \tag{6}$$

with

$$f^*_{n,t}(u) = (t_2 - \frac{u^2}{n})^{\frac{n-1}{2}-1}(t_3 - \frac{(t_1-u)^2}{n})^{\frac{n-1}{2}-1}\,;$$
$$K_1 = \alpha \int_\Omega f^*_{n,t}(u)\ du\,;$$

in which

$$U = \sum_{i=1}^n X_i\,, \quad T_1 = \sum_{i=1}^n X_i + \sum_{i=1}^n Y_i\,, \quad T_2 = \sum_{i=1}^n X_i^2\,, \quad T_3 = \sum_{i=1}^n Y_i^2\,, \quad T = (T_1, T_2, T_3)\,;$$

with $\Omega = \{u|\max(-\sqrt{nt_2}, t_1 - \sqrt{nt_3}) \le u \le \min(\sqrt{nt_2}, t_1 + \sqrt{nt_3})\}$ *to be the support of the joint density function of* (U, T).

We call the statistic U in Theorem 1 to be the mean-variance-ratio one-sided test statistic or simply the mean-variance-ratio test statistic for the hypotheses setup in (3) if no confusion occurs. Next, we introduce the UMPU statistic as stated in the following theorem to test for the equality of the mean-variance ratios listed in (4):

Theorem 2 *Let* X_i *and* Y_i $(i = 1, 2, \cdots, n)$ *be independent random variables with joint distribution function defined in (1). Then, for the hypotheses setup in (4), there exists a UMPU level-*α *test with critical function*

$$\phi(u,t) = \begin{cases} 1, & \text{when } u \le C_1(t) \text{ or } \ge C_2(t) \\ 0, & \text{when } C_1(t) < u < C_2(t) \end{cases} \tag{7}$$

in which C_1 *and* C_2 *satisfy*

$$\begin{cases} \int_{C_1}^{C_2} f^*_{n,t}(u)\ du = K_2 \\ \int_{C_1}^{C_2} u f^*_{n,t}(u)\ du = K_3 \end{cases} \tag{8}$$

where

$$K_2 = (1-\alpha)\int_\Omega f^*_{n,t}(u)\ du,$$
$$K_3 = (1-\alpha)\int_\Omega u f^*_{n,t}(u)\ du\,.$$

The terms $f^*_{n,t}(u)$, T_i $(i = 1, 2, 3)$ and T are defined in Theorem 1.

We call the statistic U in Theorem 2 to be the mean-variance-ratio two-sided test statistic or simply the mean-variance-ratio test statistic for the hypotheses setup in (4) if no confusion occurs. As equations in Theorem 2 are complicated integral equations, their exact solutions cannot be solved mathematically. Thus, we turn to look for the numerical solutions to these equations and apply numerical methods to solve equations as stated in the following problem:

Problem 3 *To compute the values of the constants C_1 and C_2 in $\Omega = [I_d, I_u]$ such that*

$$\int_{C_1}^{C_2} f^*_{n,t}(u)\ du = K_2 \tag{9}$$

and

$$\int_{C_1}^{C_2} u f^*_{n,t}(u)\ du = K_3 \tag{10}$$

where

$$f^*_{n,t}(u) = (t_2 - \frac{u^2}{n})^{\frac{n-1}{2}-1}(t_3 - \frac{(t_1-u)^2}{n})^{\frac{n-1}{2}-1}$$

$$K_2 = (1-\alpha)\int_{\Omega} f^*_{n,t}(u)\ du,$$

and

$$K_3 = (1-\alpha)\int_{\Omega} u f^*_{n,t}(u)\ du\,.$$

To solve this problem, we have to conduct the following steps:

Step 1: We first let

$$\delta_0 = (I_u - I_d)/K, \quad \text{and} \quad C_1 = I_d \tag{11}$$

where I_d and I_u are two end points of the support interval defined in Problem 3. Here, K is an integer chosen to be big enough, say for example, 400, such that $(I_u - I_d)/K$ is set to be a small increment.

Step 2: Thereafter, we let

$$C_1 = C_1 + k\delta_0, \quad k = 0,\ 1, ...,\ K.$$

For each C_1, we are going to solve equations in (9) and (10) to obtain two values of C_2's approximately, one obtained by solving (9) and another obtained by solving (10). If the values of two C_2's obtained above are approximately equal, they could be used as the approximate solutions to equations in (9) and (10). If not, we move on to let $k = k+1$ and continue the process in Step 2 till the values of two C_2's are approximately equal. In this procedure, we can achieve the appropriate calculation precision by controlling the precision of solutions to equations in (9) and (10) respectively. We note that one could choose a very large value for K so as to get δ_0 as small as possible. However, it is not necessary to do so as any big value of K could not improve the calculation precision remarkably and thus we suggest using 400 which is large enough.

3 Illustration

In this section, we demonstrate the superiority of the mean-variance tests developed in this paper over the traditional Sharpe-ratio tests by illustrating the applicability of our tests to the decision making process of investing in commodity trading advisors (CTAs). For simplicity, we only demonstrate the two-sided UMPU test.[1] The data analyzed in this section are the monthly returns of 61 indices from Commodity Trading Advisors (CTAs) for the sample period from January 2001 to December 2004 in which the data from Jan 2003 to Dec 2003 are used to compute the mean-variance ratio in Jan 2004, while the data from Feb 2003 to Jan 2004 are used to compute the mean-variance ratio in Feb 2004, and so on. However, using too short periods to compute the Sharpe ratio would not be meaningful as discussed in our previous sections. Thus, we utilize a longer period from Jan 2001 to Dec 2003 to compute the Sharpe-ratio ratio in Jan 2004, from Feb 2001 to Jan 2004 to compute the in Feb 2004, and so on.[2]

For simplicity, in our illustration we only report the comparison of three pairs

[1]The results of the one-sided test which drew a similar conclusion is available on request.

[2]We note that, actually, we should use even longer periods to compute the Sharpe ratio but the data is not available. Also, the results for too long periods are expected to yield insignificant difference for all comparison, which is not useful to investors.

of indices with the largest or smallest means, variances, or mean-variance ratios, respectively, from January 2004 and December 2004. They are: AIS Futures Fund LP (maximum mean, X_{11}) versus Beacon Currency Fund (minimum mean, denoted by X_{12}), JWH Global Financial & Energy Portfolio (maximum variance, X_{21}) versus Worldwide Financial Futures Program (minimum variance, X_{22}), Oceanus Fund Ltd (maximum mean-variance ratio, X_{31}) versus Beacon Currency Fund (minimum mean-variance ratio, X_{32}). Let $r_{ij,t}$ be the excess return of X_{ij} over the risk-free interest rate at time t with mean μ_{ij} and variance σ_{ij}^2 for $i = 1, 2, 3$ and $j = 1, 2$ respectively. The 3-month Treasury bills rate obtained from Datastream is used to proxy the risk-free rate. We test the following hypotheses:

$$H_{0i} : \frac{\mu_{i1}}{\sigma_{i1}^2} = \frac{\mu_{i2}}{\sigma_{i2}^2} \quad \text{versus} \quad H_{1i} : \frac{\mu_{i1}}{\sigma_{i1}^2} \neq \frac{\mu_{i2}}{\sigma_{i2}^2} \quad \text{for } i = 1, 2, 3. \tag{12}$$

To test the hypotheses in (12), we first compute the values of the test function U for the mean-variance ratio statistic shown in (**??**) for each pair of funds and display the values in Tables 1, 2 and 3 respectively. We then compute the critical values C_1 and C_2 under the test level of 0.05 for each pair of funds to test the hypotheses in (12). In addition, in order to illustrate the performance of the funds and their corresponding test results visually, we first exhibit the returns of the two funds being compared and their difference for each pair of funds in Figures 1A, 2A, and 3A respectively, displaying their corresponding values of U with C_1 and C_2 in Figures 1B, 2B and 3B respectively.

For comparison, we also compute the corresponding Sharpe-ratio statistic developed by Jobson and Korkie (1981) and Memmel (2003) such that

$$z_i = \frac{\hat{\sigma_{i2}}\hat{\mu_{i1}} - \hat{\sigma_{i1}}\hat{\mu_{i2}}}{\sqrt{\hat{\theta}}} \tag{13}$$

which follows standard normal distribution asymptotically with

$$\theta = \frac{1}{T}\big[2\sigma_{i1}^2\sigma_{i2}^2 - 2\sigma_{i1}\sigma_{i2}\sigma_{i1,i2} + \frac{1}{2}\mu_{i1}^2\sigma_{i2}^2 + \frac{1}{2}\mu_{i2}^2\sigma_{i1}^2 - \frac{\mu_{i1}\mu_{i2}}{\sigma_{i1}\sigma_{i2}}\sigma_{i1,i2}^2\big]$$

to test for the equality of Sharpe ratios for the funds by the following hypotheses such that

$$H_{0i}^{*} : \frac{\mu_{i1}}{\sigma_{i1}} = \frac{\mu_{i2}}{\sigma_{i2}} \quad \text{versus} \quad H_{1i}^{*} : \frac{\mu_{i1}}{\sigma_{i1}} \neq \frac{\mu_{i2}}{\sigma_{i2}} \quad \text{for } \; i = 1, 2, 3. \tag{14}$$

Different from using one-year data to compute the values of our proposed statistic, we use the overlapping three-year data to compute the Sharpe-ratio statistic for the year 2004, as this statistic is only valid asymptotically. The results are also reported in Tables 1 to 3 next to the results for our proposed statistic while their plots and their critical values are depicted in Figures 1C to 3C for comparison.

We first examine the performance between the returns of AIS Futures Fund LP, the fund with the largest mean, and that of Beacon Currency Fund, the fund with the smallest mean. As shown in Table 1 and Figure 1C, we cannot detect any significant difference between their Sharpe ratios, implying that the performances of these two funds are indistinguishable. We note that the three-year monthly data being used to compute the Sharpe-ratio statistic could still be too short to satisfy the asymptotic statistical properties for the test but we cannot find any significant difference between the performance of these two funds. If we use any longer period, the result is expected to be insignificant as the high means in some sub-periods could be offset by the low means in other sub-periods. Thus, a possible limitation of applying the Sharpe-ratio test is that it would usually conclude indistinguishable performances among the funds, which may not be the case. In this aspect, looking for a statistic to evaluate the performance among assets for short periods is essential. In this paper, we adopt our proposed statistic to conduct the analysis. As shown in Table 1 and Figure 1, we find that our proposed statistic does not disappointed us that it does show significant difference in performance between these two funds in some periods. This information will be useful to investors for their decision making process.

Similar conclusion could also be drawn for the comparison between JWH Global Financial & Energy Portfolio and Worldwide Financial Futures Program; the former is the fund possessing the maximum variance while the latter attains the minimum variance. Again, applying the Sharpe-ratio test concludes that the performance between these two funds are indistinguishable while invoking our proposed statistic enables us to detect significant differences.

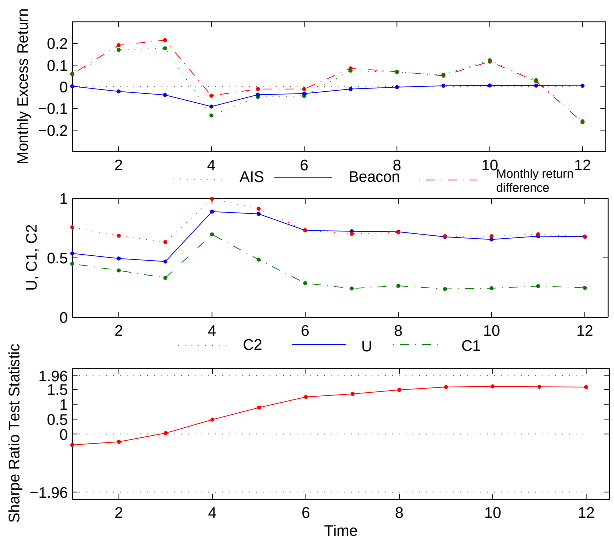
Then, we turn to investigate the performance between Oceanus Fund Ltd and Beacon Currency Fund in 2004, with the former possessing the maximum mean-variance ratio while the latter attaining the minimum mean-variance ratio. From Table 3, we find that the difference between these two funds becomes very small after June 2004 and even turn positive to negative in September 2004. However, the Sharpe-ratio test cannot detect such change and indicates that Oceanus Fund Ltd performs significantly better than Beacon Currency Fund in the entire 2004. In applying our proposed mean-variance-ratio test, this test reveals that the change in its value has become insignificant after June 2004. The information that is derived from our proposed test is thus useful for investors who takes their decision making with regard to their investment seriously.

4 Concluding Remarks

To evaluate the performance among the assets for small samples, we develop both one-sided and two-sided mean-variance-ratio statistics to test the hypothesis of the equality of mean-variance ratios between two assets in this paper. In addition, we prove that our proposed statistics are uniformly most powerful unbiased tests. We illustrate the superiority of our proposed test over the traditional Sharpe-ratio test by applying both tests to analyze the performance of funds from Commodity Trading Advisors. Our findings show that while the traditional Sharpe-ratio test concludes most of the CTA funds being analyzed as being indistinguishable in their performance, our proposed statistic shows that some funds outperform the others. In addition, when we apply the Sharpe-ratio statistic on some other funds, we find that the statistic indicates that one fund is significantly outperforming another fund even though the difference between the two funds become insignificantly small or even changes directions. However, when our proposed mean-variance-ratio statistic is applied, we could detect such changes. This shows the superiority of our proposed statistic in revealing short term performance and in return enables the investors to make better decision about their investments.

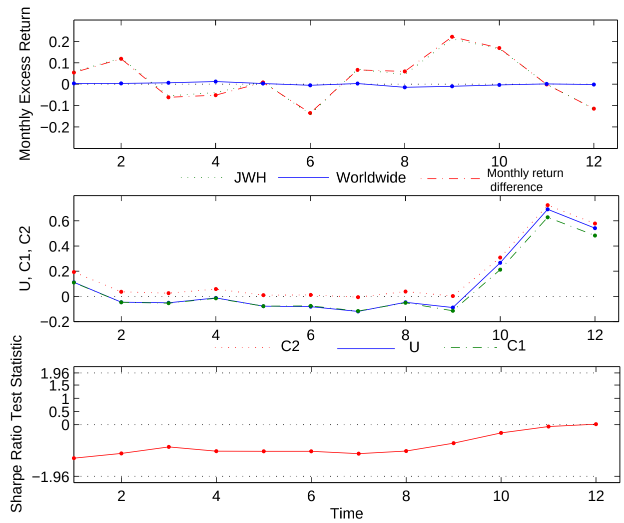
We note that our proposed test statistic could also be used in many different contexts in Finance and Economics. For example, the statistic could be used to extend the work of Levy (1972), Cumby and Glen (1990), Jorion (1991), Grinblatt and Titman (1994) and MacKinlay and Pastor (2000) in the evaluation of portfolio performance, which, in turn, could be used in risk management and in testing market efficiency which could shed new light on asset investments.

Figure 1: Plots of the monthly excess returns for AIS Futures Fund LP and Beacon Currency Fund and corresponding Mean-Variance Ratio test U and Sharpe ratio test statistic Z



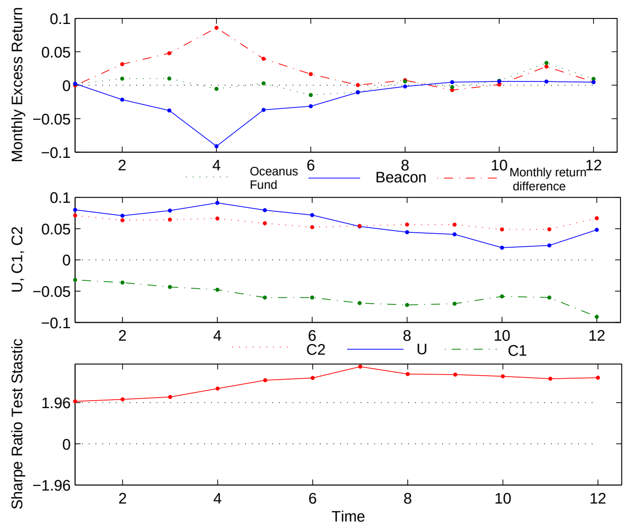
Note: The Mean-Variance Ratio test U is defined in Theorem 1 with C_1 and C_2 defined in (8) and the Sharpe ratio test statistic Z is defined in (13).

Figure 2: Plots of Monthly excess returns of JWH Global Financial & Energy Portfolio and Worldwide Financial Futures Program and corresponding Mean-Variance Ratio test U and Sharpe ratio test statistic Z



Note: The Mean-Variance Ratio test U is defined in Theorem 1 with C_1 and C_2 defined in (8) and the Sharpe ratio test statistic Z is defined in (13).

Figure 3: Plots of the monthly excess returns for Oceanus Fund Ltd versus Beacon Currency Fund and corresponding Mean-Variance Ratio test U and Sharpe ratio test statistic Z



Note: The Mean-Variance Ratio test U is defined in Theorem 1 with C_1 and C_2 defined in (8) and the Sharpe ratio test statistic Z is defined in (13).

Table 1: The Results of the Mean-Variance Ratio Test and Sharpe ratio Test for AIS Futures Fund LP versus Beacon Currency Fund in 2004

		Mean-Variance Ratio Test			Sharpe ratio Test	
Time	$X_{11} - X_{12}$	U	C_1	C_2	Z	p-value
Jan	0.0580	0.5368	0.4501	0.7574	-0.3717	0.71
Feb	0.1923	0.4943	0.3938	0.6860	-0.2634	0.79
Mar	0.2153	0.4692	0.3311	0.6315	0.0291	0.98
Apr	-0.0412	0.8881	0.6969	0.9963	0.4823	0.63
May	-0.0104	0.8691	0.4846	0.9127	0.8899	0.37
Jun	-0.0107	0.7300	0.2861	0.7310	1.2447	0.21
Jul	0.0851	0.7234*	0.2424	0.7013	1.3469	0.18
Aug	0.0697	0.7190*	0.2647	0.7130	1.4847	0.14
Sep	0.0513	0.6762	0.2381	0.6817	1.5838	0.11
Oct	0.1166	0.6545	0.2441	0.6812	1.6034	0.11
Nov	0.0251	0.6813	0.2618	0.6971	1.5911	0.11
Dec	-0.1639	0.6784*	0.2471	0.6760	1.5783	0.11

* $p < 5\%$, the Mean-Variance Ratio Test U is defined in (7) while the Sharpe ratio Test Z is defined in (13).

Table 2: The Results of the Mean-Variance Ratio Test and Sharpe ratio Test for JWH Global Financial & Energy Portfolio versus Worldwide Financial Futures Program in 2004

		Mean-Variance Ratio Test			Sharpe ratio Test	
Time	$X_{21} - X_{22}$	U	C_1	C_2	Z	p-value
Jan	0.0542	0.1125	0.1103	0.1932	-1.2710	0.20
Feb	0.1188	-0.0455	-0.0465	0.0366	-1.0933	0.27
Mar	-0.0616	-0.0507	-0.0538	0.0253	-0.8457	0.40
Apr	-0.0514	-0.0115	-0.0153	0.0581	-1.0057	0.31
May	0.0082	-0.0773	-0.0775	0.0101	-1.0128	0.31
Jun	-0.1350	-0.0814*	-0.0757	0.0116	-1.0132	0.31
Jul	0.0664	-0.1188*	-0.1157	-0.0062	-1.1002	0.27
Aug	0.0597	-0.0464	-0.0504	0.0381	-1.0017	0.32
Sep	0.2215	-0.0879	-0.1143	0.0032	-0.7042	0.48
Oct	0.1690	0.2671	0.2124	0.3086	-0.3144	0.75
Nov	-0.0011	0.6918	0.6276	0.7242	-0.0742	0.94
Dec	-0.1143	0.5415	0.4828	0.5787	0.0165	0.99

* $p < 5\%$, the Mean-Variance Ratio Test U is defined in (7) while the Sharpe ratio Test Z is defined in (13).

Table 3: The Results of the Mean-Variance Ratio Test and Sharpe ratio Test for Oceanus Fund Ltd versus Beacon Currency Fund in 2004

		Mean-Variance Ratio Test			Sharpe ratio Test	
Time	$X_{31} - X_{32}$	U	C_1	C_2	Z	p-value
Jan	-0.0003	0.0801*	-0.0320	0.0711	2.0180	0.04*
Feb	0.0315	0.0707*	-0.0362	0.0635	2.1148	0.03*
Mar	0.0477	0.0789*	-0.0434	0.0644	2.2311	0.03*
Apr	0.0858	0.0913*	-0.0478	0.0662	2.6265	0.01*
May	0.0396	0.0795*	-0.0602	0.0586	3.0249	0.00*
Jun	0.0166	0.0718*	-0.0602	0.0524	3.1306	0.00*
Jul	0.0002	0.0535	-0.0690	0.0543	3.6708	0.00*
Aug	0.0075	0.0444	-0.0722	0.0566	3.3177	0.00*
Sep	-0.0074	0.0410	-0.0701	0.0563	3.2926	0.00*
Oct	0.0010	0.0195	-0.0584	0.0488	3.2088	0.00*
Nov	0.0278	0.0231	-0.0602	0.0490	3.0957	0.00*
Dec	0.0049	0.0481	-0.0911	0.0666	3.1418	0.00*

* $p < 5\%$, the Mean-Variance Ratio Test U is defined in (7) while the Sharpe ratio Test Z is defined in (13).

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