Portfolio Optimization with Alternative Investments

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Abstract

Most monthly return distributions of alternative assets are in general not normally distributed. Furthermore, some have biases (e.g., survivorship), which can distort the risk-return profile. For that reason, every portfolio optimization in the mean-variance framework that includes alternative assets will most likely be suboptimal, as the risk-return characteristics will not be adequately covered. Our first step is to correct the return series for biases. Our next step is to replace the empirical return distributions with two normal distributions to approximate a best-fit distribution to cover the impact of the higher moments. This procedure is known as the mixture of normal method, and is widely used in financial applications. In order to build a strategic asset allocation for a mixed-asset portfolio, we consider traditional investments (stocks and bonds), and the vast majority of alternative investments (asset-backed securities, hedge funds, venture capital, private equity (buyout), commodities, and REITs). In our optimization procedure, we also consider real investor preferences. In order to test our results for stability, we apply robustness tests, which allow for the time-varying correlation structures of the strategies.

JEL Classification: G2, G12, G31

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1 Introduction

Alternative investments, which totalled U.S. $3 trillion as of the end of 2006, have become increasingly important in a portfolio context for institutional investors such as endowments and high net worth individuals [Loeys and Panigirtzoglou (2006)]. These investors enjoy regulatory freedom, have sufficient capital to invest in alternative investments like private equity or hedge funds, and have long enough investment horizons to hold illiquid investments. The share of alternative investments in the portfolios of high net worth individuals increased from 10% in 2002 to 20% in 2005 [for more details, see World Wealth Report 2006 of Capgemini and Merrill Lynch]. And their average share in an endowment portfolio increased from 3% in 1996 to 12% in 2005. Endowments with greater than U.S. $1 billion have an allocation of about 36% [for more details, see 2006 NABUCO Endowment Study].

But what factors are driving this rush to alternative investments? I argue there are two main reasons. First, investors are seeking diversification to avoid a repeat of the substantial losses they experienced during the recent equity and bond market downturns (e.g., the Asian crisis of 1997, the Russian crisis of 1998, the new economy bubble in 2000, and the World Trade Center attacks in 2001). Alternative investments are practical during more volatile market phases, because their return drivers differ from those of the equity and bond markets [Schneeweis, Kazemi and Martin (2001)].

Second, the positive diversification properties of alternative investments do not reduce their expected portfolio returns, and may enhance risk-adjusted performance. For example, the top U.S. university endowments (e.g., Harvard, Princeton, and Yale) reported realized annual returns of 10%-25% over the last three years, which highlights that alternative investments can enhance expected portfolio returns too. Lerner, Schoar and Wang (2007) attribute part of this success to willingness to rely on alternative investments.

However, if investors want to build an exposure to alternative investments, they must determine which investments to include, as well as the strategic asset allocation. Because strategic asset allocation explains most of the portfolio return variability, it is the major determinant of investment performance and the most critical decision in the investment process [Hoernemann, Junkans and Zarate (2005)[1]].

Investors must also consider the risk-return characteristics, because they are the primary influence on the strategic asset allocation models. The model of choice must be flexible enough to incorporate the risk-return characteristics. If they are not captured properly, or if the strategic asset allocation model is not flexible enough, the obtained optimal portfolio may only include the alternative investments [Terhaar, Staub and Singer (2003)].

The majority of studies in the literature only focus on the effects of including one alternative investment in a mixed-asset portfolio. When more than one is included, the risk-return profiles are often not captured adequately, or the chosen model is not flexible enough [e.g. Schneeweis, Karavas and Georgiev (2002) and Conner (2003)]. Alternatively, the alternative investments may not be representative of the entire universe. For example, Huang and Zhong (2006) consider commodities, REITs, and TIPs; Hoecht, Ng, Wolf and Zagst (2007) integrate only Asian hedge funds and Asian REITs. These papers do not provide a strategic asset allocation for a broad sample of alternative investments.

[1] The authors present an alternative to the oft-cited studies of Brinson, Hood and Beebower (1986) and Brinson, Hood and Beebower (1991). They use a slightly different framework and covers a longer time horizon. It also includes alternative assets and uses synthetic portfolios.

To our knowledge, this paper is the first that 1) incorporates a variety of alternative investments (e.g., asset-backed securities, hedge funds, venture capital, private equity (buyout), commodities, and REITs) and traditional investments (stocks and government bonds), 2) adjusts risk-return profiles to account for biases, 3) uses a strategic asset allocation model that is flexible enough to capture the risk-return profile adequately, and 4) incorporates real investor preferences.

Before the optimization, the return time series of some alternative investments (hedge funds, venture capital, buyout) are corrected for biases such as appraisal smoothing, stale pricing and/or survivorship. The optimization is thus flexible enough to incorporate any potential risk arising from higher moments (skewness and kurtosis) that would not be covered by the standard deviation. This is important, because the empirical return distributions of some alternative investments are generally not normally distributed. Thus every portfolio optimization in the mean-variance will likely be suboptimal.

Consequently, we use the mixture of normal method to replace the empirical return distributions (which often exhibit skewness and positive excess kurtosis) with two normal distributions to approximate a best fit distribution. This approach ensures that the best fit return distributions exhibit the higher moments close to their empirical pendants. We then use the best fit distributions in the optimization procedure. To derive the strategic asset allocation, we apply a goal function so that we can examine real investor preferences for risk aversion. As a robustness check, we test our results for time-varying correlations.

Our general findings are that U.S. government bonds, venture capital, buyout, and REITs have the largest portfolio weights in the optimal portfolios. The importance of private equity-related asset classes and REITs decreases with increasing risk aversion. In contrast to the private equity-related asset classes and REITs, U.S. government bonds increase in importance when risk aversion increases. U.S. equity (5%-8%) and commodities (3%-4%) have relatively constant portfolio weights regardless of the degree of risk aversion. Hedge funds are included only in the high and low risk aversion regimes, with portfolio weights of 1%-2%. In contrast, asset-backed securities in the medium risk aversion portfolios have weights of between 1% and 8%. In conclusion, we find that alternative investments are important for the strategic asset allocation of institutional investors such as endowments, family offices, pension funds, and high net worth individuals who have sufficient time horizons and investment capital. However, not all alternative investment classes are of equal importance. Alternative investments are not appropriate as substitutes for traditional asset classes, and may be better as complements for achieving the desired risk-return profiles.

The rest of this paper proceeds as follows: Section II evaluates which alternative investments should be included in a mixed-asset portfolio. We review current literature where at least one alternative investment is integrated into a mixed-asset portfolio. Section III presents the proxy indices that we consider for the traditional and alternative investments and for the bias adjustment. Section IV explains the methodology of our research design, and in section V we discuss our results. Section VI concludes, with a summary, discussion, and implications for future research.

2. Evaluation of Alternative Investments in Mixed-Asset Portfolios

It has been well-known since Markowitz's (1952) seminal paper on portfolio theory that diversification can increase portfolio expected returns while reducing volatility. However, investors should not blindly add another asset class to their portfolios without carefully considering its properties in the context of the portfolio. A naively chosen allocation to the newly added asset class may not improve the risk-return profile, and may even worsen it. This raises the question of whether alternative investments really improve the (risk-adjusted) performance of a (mixed-asset) portfolio, and whether they should be included in the strategic asset allocation.

Following Kat (2007), we analyze four asset classes – commodities, hedge funds, private equity (buyout, venture capital) and REITs – to determine their ability to enhance the risk-return profile of an existing traditional portfolio (stocks and bonds).[2] We also discuss potential biases in the return time series that can affect the risk-return profile, and select a proxy index that best represents the characteristics of each asset class.

2.1 Commodities

In order to study the risk-return characteristics of commodities, our first step is to determine which exposure best reflects their price behaviour. There are several ways to participate in commodity markets via a number of different kinds of financial instruments. The most important are: 1) direct investment in the physical good, 2) indirect investment in stocks of natural resource companies, 3) commodity funds, or 4) investment in commodity futures indices.

Direct physical investment is generally not practical, because most commodities are perishable. Geman (2005) notes, however, that precious metals like gold, silver, and platinum are an exception. They do not have high current costs and are not difficult to store. However, a portfolio consisting solely of precious metals would not be sufficiently diversified. And, as Till and Eagleeye (2005) find, commodities that are difficult to store have higher expected returns than those that are not.

[2] We do not discuss asset-backed securities in detail because few studies have analyzed their risk-return characteristics. However, Funke, Johanning and Michel (2005) and Benk and Johanning (2008) have found that asset-backed securities can be a valuable addition to institutional investor portfolios.

Indirect investment in commodities via commodity stocks is an insufficient substitute for direct investment. Listed commodity stocks have their own characteristics and risks, so investors do not receive direct commodity exposure. Fabozzi, Fuess and Kaiser (2007) note that the major sources of varying movements between commodity stocks and the underlying commodity are: operational risk caused by human or technical failure, internal regulations, external events, the strategic position of the company, management quality, capital structure (the debt/equity ratio), the expectations and ratings of company and profit growth, risk sensitivity, the risk of a total loss if prices decrease below total production costs, information transparency, information credibility, and temporary mispricing due to market disequilibriums. Furthermore, Georgiev (2006) shows that these sector-specific stocks are only slightly correlated with commodity prices.

Investing in a portfolio consisting of commodity stocks via a commodity fund can be either active or passive. With a passively managed fund, the same discrepancies in the risk-return characteristics of the underlying commodity and commodity stock will apply. With an actively managed commodity fund (e.g., a commodity trading advisor [CTA]), there is additional distortion from the fund manager’s skill [Gregoriou and Rouah (2004), Akey (2006) and Idzorek (2006)].

In summary, none of the abovementioned methods of obtaining commodity exposure measures the risk-return characteristics adequately. For the majority of investors, an index-oriented investment will be the most efficient [Fabozzi, Fuess and Kaiser (2008)]. Therefore, investable commodity futures indices are the best available proxies for the risk-return profile of commodities, and most studies use these indices as benchmarks for the development of the commodity markets. The most widely used are the CRB/Reuters Commodity Index, the S&P GSCI Commodity Index, and the Dow Jones-AIG Commodity Index.[3]

But whether investors in commodities earn a risk premium is an ongoing discussion. Early studies such as Bodie and Rosansky (1980), Kaplan and Lummer (1998), and Greer (2000), as well as a more recent study by Gorton and Rouwenhorst (2006), find that historical returns of unleveraged commodity futures indices equal stock market returns. In contrast, Erb and Harvey (2006) find decreasing returns over time, and less evidence of a significant return persistence for single commodity sectors.[4] Kat and Oomen (2007) also find no evidence of a consistent risk premium, except for energy commodities.[5]

Thus the existence of a risk premium for single commodities is still a contentious issue. Nevertheless, structuring a commodity portfolio will only gain in importance for investors, because even without a risk premium, a well diversified portfolio of commodities is assumed to offer a reliable source of return (Erb and Harvey (2006) and Scherer and He (2008) refer to this as the diversification return).

In contrast to the risk premium controversy, there is a consensus in the literature that investable commodity futures indices exhibit positive properties in diversifying mixed-asset portfolios [see, for example, Bodie and Rosansky (1980), Kaplan and Lummer (1998), Anson (1999), Jensen, Johnson and Mercer (2000), Gorton and Rouwenhorst (2005), Georgiev (2006),

[3] The S&P GSCI Commodity Index, for example, quintupled in size from 2002 to March 2007 to U.S. $60 billion. It is estimated that as of March 2007, about U.S. $90 billion was invested in commodity futures indices, almost seven times the amount invested in 2002 [Doyle, Hill and Jack (2007)]. At the beginning of 2007, Standard & Poor’s acquired the GSCI Commodity Index, which was subsequently renamed the S&P GSCI Commodity Index.

[4] Erb and Harvey (2006) argue that a commodity futures index is not necessarily a good measure of aggregate commodity market performance, because part of the excess return is due to rebalancing. The resulting rebalancing bonus (also called volatility pumping) is described by Fernholz and Shay (1982).

[5] Energy is considered a subgroup of the commodities natural gas, crude oils, unleaded gasoline, and heating oil.

Gordon (2006), Idzorek (2006), Fabozzi, Fuess and Kaiser (2008), and Scherer and He (2008), among others].

Thus investable commodity futures indices are the most appropriate way to build an exposure to commodities that can capture the risk-return characteristics of the commodity market. It is not clear whether the underlying commodities of a commodity futures index generate a risk premium, but there is strong evidence that an index can earn a diversification return and exhibit positive properties in diversifying mixed-asset portfolios. So we include here the S&P GSCI Commodity Total Return Index,[6] which has the highest level of invested capital.

2.2 Hedge Funds

Fung and Hsieh (2000), Amenc, Curtis and Martellini (2003), and Fuess, Kaiser and Adams (2007) summarize the most important reasons for investing in hedge funds. The first is that hedge fund managers enjoy greater flexibility to generate positive "alpha returns," because hedge funds are subject to much less regulation. The second reason is that hedge fund returns exhibit low correlations with traditional asset classes (in other words, the risk premiums of hedge fund strategies generally exhibit low correlations with equity or fixed-income risk premiums.[7] By using allocations to hedge funds, investors can diversify their traditional long-only portfolios.

The academic literature holds numerous examples of empirical studies on the performance persistence of hedge fund managers, but with mixed results. Agarwal and Naik (2000a), Agarwal and Naik (2000b), and Jagannathan, Malakhov and Novikov (2007) find evidence of performance persistence; however, Brown, Goetzmann and Ibbotson (1999) do not. The different conclusions

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[6] Fabozzi, Fuess and Kaiser (2008) discuss in detail the different calculation methods spot, excess and total return of the S&P GSCI Commodity Index.

[7] Hedge fund managers are not constrained by short-selling and are allowed to invest in other non-traditional asset classes like credit derivatives, mortgaged backed securities, etc. in a way that is not allowed to mutual fund managers.

can be attributed to the use of different databases, sample periods, performance measures, and statistical methodologies.[8]

Even if hedge fund managers do not generate a persistent alpha return, their returns show low correlation with those of traditional asset classes. Before assessing the diversification potential of hedge fund investing, however, we need to examine the higher moments of the return distribution of hedge fund returns. Fung and Hsieh (1997), Fung and Hsieh (2000), Amin and Kat (2003a), and Brooks and Kat (2002), among others, note that the return distributions of hedge funds and hedge fund indices are not normally distributed, and exhibit skewness and excess kurtosis. The return distributions also show positive first-order autocorrelation due to illiquidity [Avramov, Kosowski, Naik and Teo (2007)]. This autocorrelation causes estimates of the standard deviation of hedge fund returns to exhibit a systematic downward bias [Kat (2003)].[9]

For these reasons, the mean-variance framework is not adequate to cover the characteristics of hedge fund returns [see, for example, Fung and Hsieh (1997) and Fung and Hsieh (1999)]. However, if we apply the mean-variance framework, the integration of hedge funds into mixed-asset portfolios improves the mean-variance properties, but leads to significantly lower skewness and higher kurtosis in the portfolios on the efficient frontier [Amin and Kat (2002), Amin and Kat (2003b), and Lhabitant and Learned (2002)]. The implication that these risks occur as hedge funds are introduced into a mixed-asset portfolio, however, must be considered.

Kat (2005) emphasizes the use of multi-moment optimizations to account for skewness and kurtosis. Kooli, Amvella and Gueyie (2005), Favre and Galeano (2002), Favre and Signer (2002), and Lamm (2003) try to incorporate this issue by replacing the risk measure variance with, for

[8] See Eling (2007) for a survey.
[9] The major reason for the positive autocorrelation is illiquid trading strategies [Lo and McKinlay (1990)].

example, Cornish-Fisher value at risk,[10] mean-target semideviation, and other risk measures that incorporate the downside risk of hedge funds. Agarwal and Naik (2004), in contrast, favour the risk measure conditional value at risk, which has several advantages over value at risk and is a coherent risk measure, as noted by Artzner, Delbaen, Eber and Heath (1999).

Other researchers have chosen not to change the risk measure, investigating instead the hedge fund allocations in different frameworks. Examples include Amenc and Martellini (2003), who investigate several models that allow investors to quantitatively estimate the optimal portfolio weight for hedge funds, Jurczenko, Maille and Merlin (2005), who use a shortage function, and Davies, Kat and Lu (2006), who use the polynomial goal programming technique. All of these models include the moments of the return distribution until the fourth moment. Popova, Morton, Popova and Yau (2007) apply a target function and incorporate the higher moments of the hedge fund return distribution with the mixture of two normal distributions.

In summary, we find that the more consideration is given to the inherent characteristics of hedge fund return distributions in the models, the lower are the resulting weights, in general. But the weights in mixed-asset portfolios are still substantial, and the risk-return profile of the resulting portfolios is enhanced.[11]

Thus, hedge fund return distributions often exhibit positive autocorrelation and contain additional sources of risk not considered with the normal distribution (e.g., negative skewness

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[10] The Cornish-Fisher value at risk, developed by Cornish and Fisher (1937), is an extension to the value at risk concept that adjusts for higher moments in the return distribution (e.g., skewness and kurtosis). Kaplanski and Kroll (2002) have criticized the value at risk concept as an inadequate measure within the expected utility framework. Another drawback results if the returns are not normally distributed. The estimation of mean-value at risk efficient portfolios can become very difficult, especially if the return distribution is discrete. In this case, the frontier estimated by the value at risk dependent on the portfolio weights is non-convex, non-smooth, and has multiple local extrema [Gaivoronski and Pflug (2005) and Rockafellar and Uryasev (2002)].

[11] Gueyie and Amvella (2006) and Kooli (2007) show that investment in funds of hedge funds, the typical beginning investment in this asset class, can improve the risk-return profile of a portfolio. Belvedere (2001) and Favre and Galeano (2001) suggest that hedge fund investing is favourable for pension funds.

and positive excess kurtosis). When these inherent characteristics are included in the models and/or in the risk measure, the portfolio weights in mixed-asset portfolios are generally lower. However, the resulting weights in the mixed-asset portfolio for hedge funds can still help diversify the portfolio, despite mixed evidence on the performance persistence. For that reason, a representative hedge fund proxy index should be included. There are many available, differing widely in calculation method and classification criteria. We use the Credit Suisse/Tremont Hedge Fund Index for our study, following Eling (2006), who notes that it is the most widely used index in academic works.

2.3 Private Equity

Studies that attempt to quantify the risk-return characteristics of private equity are rare in the literature, because the low transparency of private equity markets and the resulting insufficient available data hinder an aggregate comparison with other asset classes [Schmidt (2004)]. Furthermore, the target companies of private equity funds are generally not traded in a permanent marketplace with quoted prices, so most investments exhibit low liquidity. Investments in the target companies usually also come with high transaction costs, as a result of less publicly available information [Kaserer and Diller (2004)]. Empirical research in private equity is generally based on reported cash flows and the appraised values of unrealized investments[12] in order to calculate returns.

Gompers and Lerner (1997), Moskowitz and Vissing-Jørgensen (2002), Quigley and Woodward (2002), Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), Gottschalg,

[12] The resulting estimated return series by VentureXpert from Venture Economics and Cambridge Associates usually shows a positive autocorrelation. This can be explained by the composition of the reported return in growth in net asset value (NAV) for unrealized investments and the internal rates of return (IRRs) for realized investments. The positive autocorrelation of the appraised values are primarily due to the illiquidity of private equity investments and the managed pricing of the general partners of the private equity funds [Anson (2002)].

Phalippou and Zollo (2005) (who updated Kaplan and Schoar (2005) with superior information), Cochrane (2005), and Phalippou and Gottschalg (2007) have all attempted to quantify the risk-return characteristics of private equity on a fund level or on an individual portfolio company level. The results in these papers are heterogeneous due to differences in time series, data, and calculation methods. A shortcoming of most of these studies is that they rely too heavily on data provided by data vendors, which is self-reported and subject to selection bias. They are also based on unrealized and realized investments, which can introduce noise and potential biases.

Exceptions are Gompers and Lerner (1997), Ljungqvist and Richardson (2003), Kaplan and Schoar (2005) and Gottschalg, Phalippou and Zollo (2005), who provide information about the exact timing of the investments, the distribution of cash flows to investors, and the types of companies contained in each fund's portfolio. However, they examine relatively small sample sizes of 1, 73, 746, and 983 funds, respectively, compared to a universe of at least 3,500 funds.

Phalippou and Gottschalg (2007) extended their data set to 1,328 mature funds, and conclude that the stunning growth in the amount allocated to private equity cannot be attributed to genuinely high past net performance. However, even without high past returns, the return drivers for private equity might differ from those of traditional asset classes, and they might have positive correlation benefits.

Unfortunately, the aforementioned studies concentrate on determining the risk-return profiles, not the possible positive diversification attributes of private equity investments in mixed-asset portfolios. One of the first studies to consider private equity in the strategic asset allocation is Lamm and Ghaleb-Harter (2001), who find that investors should allocate between 19% and 51% to private equity. This recommendation is appropriate under a variety of alternative conditions and conservative assumptions regarding future performance. A later study by Chen, Baierl and

Kaplan (2002) points out the low correlation coefficient of 0.04 between venture capital and public stocks. Because of its relatively low correlation with stocks, an allocation to venture capital between 2% (for the minimum-variance portfolio) and 9% (in the maximum Sharpe ratio portfolio) is warranted for mixed-asset portfolios. Schmidt (2004) recommends a wide range of optimal portfolio weightings, between 3% and 65%, for minimizing mixed-asset portfolio variance or maximizing performance ratios.

A more recent study, Ennis and Sebastian (2005), proposes differentiating portfolio allocations for different types of investors. The authors conclude that thoughtful investors may want to exclude private equity altogether, while for others a few percentage points may be most appropriate. Ennis and Sebastian (2005) suggest that only moderate-size, equity-oriented funds with exceptional private equity investment skill, strong board-level support, and adequate staff resources should consider allocations of 10% or more. Admittedly, they use the Venture Economics Post-Venture Capital Index as a proxy for private equity, which does not adequately capture the risk-return profile of private equity for the purposes of the term "private."[13]

All the abovementioned studies highlight the relevance of private equity in diversifying mixed-asset portfolios, and recommend positive allocations in private equity in the strategic asset allocation. For that reason, we include venture capital and buyouts as the traditional components of private equity in our study [Ennis and Sebastian (2005)].[14]

In order to include private equity in this study, though, we must select a representative proxy index. Because the most well-known databases of Venture Economics and Cambridge Associates

[13] Weidig, Kemmerer and Born (2005) analyze the risk-return profiles of funds of private equity funds, and suggest that investments in those funds also improve the risk-return profile.

[14] We could argue that the returns of venture capital and buyouts are generated by the same drivers, and therefore only one representative of private equity needs to be included. In unreported results, however, we tested both indices for cointegration, and found no cointegration relationship. Therefore, we include both venture capital and buyouts.

have several disadvantages, we use indices from the CEPRES[15] database – CepreX US Venture Capital and CepreX US Buyout. The database is based on (partially) audited reports and precise cash flow information from private equity investment funds. This enables accurate financial calculations, and does not have the same shortcomings of non-audited reported filings of investment firms. A further advantage is that the indices are transaction-based and available monthly. The return time series cannot be reported contemporaneously, however, because it lacks sufficient transactions per month to report a reliable return. Therefore, the last reported return was at least twelve months earlier.

2.4 Real Estate Investment Trusts (REITs)

The last asset class we include is real estate, which offers several methods of participation. We discuss the differences between investing in REITs and in direct real estate to determine whether REITs are an appropriate proxy for real estate. Before adding REITs to a mixed-asset portfolio, it is critical to answer the following questions: 1) Are REITs as a proxy for the real estate market cointegrated with the direct real estate market? and 2) because REITs are publicly traded and may exhibit similar movements to the stock market, do they have a different enough risk-return profile to be considered a non-redundant asset class? If the answer to both questions is yes, REITs should offer diversification benefits to mixed-asset portfolios.

Several researchers have studied whether REITs are integrated with the direct real estate market. If they are, common factors would affect both return series, and thus both will eventually adjust to equilibrium. Gyourko and Keim (1992) find a relationship between real estate stock portfolio returns and returns of an appraisal-based direct real estate index after controlling for

[15] Cumming, Schmidt and Walz (2004), Cumming and Walz (2004), Schmidt (2004) and Schmidt, Steffen and Szabo (2007) give detailed insights about index construction and composition.

persistence in the appraisal return series. They conclude that the stock market reflects information about direct real estate markets that is later embedded in infrequent property appraisals.

Myer and Webb (1993) find that REIT index returns “Granger”-cause direct real estate returns for most of the real estate indices; Barkham and Geltner (1995) show evidence of a lagged price information transmission from the REIT markets to the direct real estate markets. Clayton and MacKinnon (2000) show that the relationship between REITs and direct real estate increased in the 1990s, which is a valuable insight, because our sample time period starts in the 1990s. They also conclude that, in the short term, both REITS and direct real estate have a place in optimal portfolios. But in the long term, only one should be included, because one is a substitute for the other.

To determine whether REITs behave like stock markets, Li and Wang (1995) and Ling and Naranjo (1999) use an integration approach. Oppenheimer and Grissom (1998), using spectral analysis, find that REITs are integrated with the stock market and that stock markets are the dominant influence on REIT returns. Interestingly, Clayton and MacKinnon (2000) come to a similar conclusion, but find that the sensitivity of REIT returns to the stock market declined significantly during the 1990s. They attribute this to the growth and maturity of the REIT market.

Since a great deal of evidence shows that both markets exhibit similar characteristics, we next show that REITs have their own risk-return profile, and are therefore non-redundant. Liang and McIntosh (1998) have shed light on this topic by using the classical style analysis of Sharpe. They find that REITs are a “unique” asset class. Chiang and Lee (2002) extend this work, and find that the price behaviour of REITs is unique and cannot be satisfactorily replicated by combining stocks, fixed-income securities, and direct real estate.

Stevenson (2001a) finds similar result for the U.K., while Saunders (1997) highlights the fact that the index weights that help explain REIT returns are not stable over time. Clayton and MacKinnon (2000) note that the correlation of REIT returns to stock markets is not stable over time. This provides additional evidence that REITs should be considered a "unique" asset class.

Our next step is to review the ability of REITs to diversify a mixed-asset portfolio. The evidence is mixed. Older work by Kuhle (1987) found no significant performance benefits of REITs in a stock portfolio. Mueller, Pauley and Morrell (1994), however, found that REITS were a valuable addition to a multi-asset portfolio, depending on the time period. Mull and Soenen (1997) show similar findings regarding time dependence.

More recent studies such as NAREIT (2002), Hudson-Wilson, Fabozzi, Gordon and Giliberto (2004), and Lee and Stevenson (2005) find that REITs offer considerable benefits to the portfolio performance of mixed-asset portfolios. Lee and Stevenson (2005) highlight the significant role REITs can play in diversification over different time horizons and holding periods. In the studies of Chen, Ho, Lu and Wu (2005) and Chiang and Ming-Long (2007), spanning tests are applied to test whether the additional inclusion of REITs to an existing portfolio enhances the efficient frontier. Both articles find that REITs can enhance the efficient frontier, and therefore should be included in a mixed-asset portfolio. The mixed results in the earlier studies might be interpreted, as per Clayton and MacKinnon (2000), that REIT markets have grown and matured over time into an asset class of their own.

To summarize, strong evidence is found that REITs are an appropriate proxy for real estate and offer risk-return characteristics that cannot be sufficiently replicated by other asset classes. These risk-return characteristics have been found to enhance the efficient frontier of multi-asset portfolios, and the resulting positive effects are well documented in the literature. Thus, we include REITs here, with the proxy index FTSE/NAREIT Equity REITS - Total Return Index, which is the most representative REIT index for the U.S.[16]

3. Data Set Description

In the previous section, we discussed in detail the characteristics of the asset classes we study here, and highlighted any potential biases. We use two traditional asset classes (proxy indices are in parentheses): U.S. equity (S&P 500 Composite - Total Return Index) and U.S. government bonds (JPM United States Government Bond - Total Return Index), and five alternative asset classes: asset-backed securities (IBOXX Coll. ABS - Total Return Index), buyouts (CepreX US Buyout), commodities (S&P GSCI Commodity Total Return Index), hedge funds (Credit Suisse/Tremont Hedge Fund Index), real estate investment trusts (FTSE/NAREIT Equity REITS - Total Return Index), and venture capital (CepreX US Venture Capital).[17] All the time series in our investigation are on a monthly basis with a January 1998 inception date, because all the indices report data from this date on. The end date for the time series is July 2006, because the indices for buyout and venture capital are transaction-based.

Table 2 reports the descriptive statistics for the raw time series that are partially biased. To obtain an unbiased data set, we correct the time series in the following three steps:

1) The literature has many examples of studies of the survivorship bias of hedge fund indices. The studies use varying sample periods, calculation methods, and databases, and the resulting survivorship bias ranges from 0.16% [Ackermann, McEnally and Ravenscraft

[16] An argument could be made for including REITs and direct real estate. Mueller and Mueller (2003) and Feldman (2004) find positive effects from adding REITs to a mixed-asset portfolio that already consists of direct real estate. Stevenson (2001b), however, does not find consistently positive effects. Because of this mixed evidence and because REITs and direct real state are integrated, we only use one proxy for real estate.

[17] Table 1 gives detailed descriptions of the proxy indices.

(1999)] to 6.22% [Liang (2002)]. However, most researchers usually put survivorship bias at 2% to 3%.[18] For that reason, the mean return for hedge funds is reduced by 2.5% p.a.

2) The time series of the S&P 500 generated a comparatively low mean return of 0.5% per month in our sample period, which is comparable to the mean return of U.S. government bonds (see Table 2). It seems improbable that stocks would generate bond-like returns in the long run with a standard deviation that is more than three times higher. Furthermore, the S&P 500 has the only mean return that is more than 50% below the long-term mean return. The means of the other time series are in the range of 20% around the long-term mean return. Thus, the mean return of the S&P 500 is increased by 5% p.a.

3) As we mentioned previously, hedge fund time series often display positive autocorrelation due to, e.g., illiquid portfolio positions. In contrast, transaction-based indices such as the buyout and venture capital indices generally exhibit negative autocorrelation. To test for first-order autocorrelation, we apply the portmanteau-test of Ljung and Box (1978). Table 3 reports significant first-order autocorrelation for the buyout, CS/T hedge fund index, and venture capital return time series. We use Geltner (1991) method to adjust for autocorrelation that distorts the standard deviation.[19]

After adjusting for the abovementioned biases, Table 4 gives the resulting descriptive statistics. Note that buyout has both the highest mean return (2.81%) and the highest standard deviation (8.79%), followed by venture capital with a mean return of 2.01% and a standard deviation of 6.49%. Commodities show a similar standard deviation (6.38%) to venture capital, but have a lower mean return of 1.01%.

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[18] See, for example, Anson (2006), Brown, Goetzmann and Ibbotson (1999), and Fung and Hsieh (2000).

[19] This method is adapted from the real estate finance literature, where due to smoothing in appraisals and infrequent property valuations, the returns of direct property investment indices display positive autocorrelation. See Geltner, MacGregor and Schwann (2003) for an overview of this topic.

U.S. equity and REITs have similar standard deviations of 4.5% and 4.12%, but different mean returns of 0.86% and 1.1%. Asset-backed securities, hedge funds, and U.S. government bonds have similar return levels (0.57%, 0.55%, and 0.45%), but the corresponding risk for these asset classes, as measured by standard deviation, is somewhat unexpected. Asset-backed securities have the highest mean return and the lowest standard deviation of 1.35%, followed by U.S. government bonds with 1.42% and hedge funds with 2.5%.

The higher moments (skewness and kurtosis) are additional potential sources of risk. REITs exhibit the lowest skewness of -0.6505, followed by U.S. government bonds (-0.5176) and U.S. equity (-0.4786). A skewness of around zero is found for hedge funds (-0.0452), asset-backed securities (0.0346), and commodities (0.0545). A positive, high skewness is found for the private equity-like asset classes venture capital (0.2269) and buyout (1.8119).

The excess kurtosis for most asset classes is close to zero. Commodities have a negative excess kurtosis of -0.4204, while asset-backed securities (0.3737), U.S. equity (0.4796), venture capital (0.5350), and U.S. government bonds (0.6741) have a slightly positive excess kurtosis. The excess kurtosis of 1.0580 for REITs is considerable, but it is not as high as for hedge funds (6.2968) and buyout (7.9188), which indicates that a substantial probability mass lies on the tails of the return distribution compared to the normal distribution.

Analyzing the higher moments of the return distribution for the asset classes shows that most return distributions do not follow a normal distribution. The Jarque-Bera test rejects the null hypothesis of a normally distributed return distribution for hedge funds, REITs, buyouts at the 1% level, U.S. government bonds at the 5% level, and U.S. equity at the 10% level. The null hypothesis is not rejected for asset-backed securities, commodities, and venture capital. Thus, relying on a mean-variance framework and ignoring the higher moments will not adequately capture the risk-return profile.

Table 5 provides insight into the diversification potential of each asset class. Venture capital has a high diversification potential, because the correlation to all other asset classes is negative except for asset-backed securities. Buyout has a similarly high diversification potential. It also has a negative correlation to all other asset classes except U.S. equity and commodities. U.S. equity has a negative correlation to the fixed-income-related asset classes U.S. government bonds and asset-backed securities, as well as to buyout. The remaining correlations are positive.

REITs exhibit similar behaviour to U.S. equity, although the correlation to U.S. government bonds is higher. However, the correlation to private equity-related asset classes is lower, and it is negative for buyout. Hedge funds have a negative correlation to the fixed-income-related asset classes U.S. government bonds and asset-backed securities, as well as to the private equity-related asset classes buyout and venture capital. Positive correlations are found for commodities, U.S. equity, and REITs.

Commodities have a low correlation to all asset classes and a negative correlation to asset-backed securities and to venture capital. The fixed-income-related asset classes U.S. government bonds and asset-backed securities are highly correlated, but only exhibit the same correlation coefficient for hedge funds and U.S. equity. The correlation coefficients show the opposite sign for all other asset classes.

After reviewing the descriptive statistics of the return distributions, we cannot determine a priori that one asset class is a substitute for another. Therefore, we must consider all the asset classes for the portfolio construction. In order to create optimal investor portfolios, our model must consider the characteristics of the asset classes adequately. We present our framework for optimal portfolio construction in the next section.

4. Methodology

We have discussed the characteristics that are unique to the different alternative asset classes and potential biases. We also concentrated on correcting these biases from the return series and discussing their statistical properties. The resulting return distributions are generally not normally distributed, and exhibit skewness and excess kurtosis.

Before using advanced strategic asset allocation models one should check weather Markowitz's (1952) mean-variance framework is sufficient. Scott and Horvath (1980) argue if one of the following conditions holds, the mean-variance framework will fail to deliver efficient results: 1) the return distributions are asymmetric, 2) the investor's utility function is of a higher order than quadratic, or 3) the mean and the variance do not completely determine the distribution. In other words, when using the mean-variance approach, we implicitly assume that either the investor's utility function is quadratic, or the return distribution of the assets is multivariate normal.

Note from Table 4 that the return distributions of most of the asset classes we consider here do not follow a normal distribution. Consequently, if we use mean-variance optimization and ignore the higher moments, the obtained portfolios will generally overallocate alternative investments.

To capture the higher moments, the literature offers a number of alternative distributions to the normal distribution. The multivariate Student t-distribution is well-suited for fat-tailed data, but it does not allow for asymmetry. The non-central multivariate t-distribution also has fat tails and is skewed. However, the skewness is linked directly to the location parameter, making it somewhat inflexible. The lognormal distribution has been used to model asset returns, but its skewness is a function of the mean and variance, not a separate parameter.

To capture higher moments adequately, we really need a distribution that is flexible enough to fit the skewness and the kurtosis. We use a combination of two different geometric Brownian motions to generate a mixture of normal diffusions. The normal mixture distribution is an extension of the normal distribution, and has been successfully applied recently in many fields of finance literature. Alexander and Scourse (2004) and Venkatramanan (2005), for example, have used this distribution to model asset returns and study option pricing problems. Venkataraman (1997) applied this concept to risk management, while Buckley, Saunders and Seco (2008), Morton, Popova and Popova (2006), Popova, Morton, Popova and Yau (2007), and Kaiser, Schweizer and Wu (2008) have also used it in asset allocation problems.

We choose the normal mixture distribution primarily for its flexibility and its tractability. In particular, let $f_1(x,\mu_1,\sigma_1)$ denote the probability density function of the first normal distribution, with mean μ_1 and standard deviation σ_1, and let $f_2(x,\mu_2,\sigma_2)$ denote the probability density function of the second normal distribution. The empirical distribution of non-normally distributed return distributions can then be approximated by a new distribution with the following probability density function:

$$\begin{aligned} f(x,\mu_1,\sigma_1,\mu_2,\sigma_2) &= 0.2\cdot f_1(x,\mu_1,\sigma_1)+0.8\cdot f_2(x,\mu_2,\sigma_2) \\ &= 0.2\cdot\frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\left(-\frac{(x-\mu_1)^2}{\sigma_1^2}\right)+0.8\cdot\frac{1}{\sqrt{2\pi\sigma_2^2}}\exp\left(-\frac{(x-\mu_2)^2}{\sigma_2^2}\right) \end{aligned} \qquad (1)$$

The economic justification is as follows. Consider a regime-switching model with two economic states: the usual and the unusual. The usual state exists 80% of the time, when a return is achieved with the distribution given by the second normal density; the unusual state exists 20% of the time, when the return is achieved with the other normal distribution.

Note we do not specify whether the unusual return is better than the usual return in terms of having a higher mean and/or lower volatility. Indeed, the unusual return could be better, worse, or the same. The latter case harks back to the classic assumption that returns are unconditionally normal. In general, this setting allows for conditionally normal returns, but unconditional returns need not be normal.

This specification offers many advantages. First, there are four free parameters: μ_1, σ_1, μ_2, and σ_2, so the first through fourth moments of the empirical distribution can be matched exactly. The skewness and excess kurtosis can also be captured. Second, with the normal density function, the new approximating distribution is still tractable. And third, as we noted before, this specification treats the traditional normal approximation as a special case. Figure 1 provides an illustration of this method.

Because the approximating parameters μ_1, σ_1, μ_2, and σ_2 cannot be solved analytically, we solve them numerically. The goal is to approximate the empirical distribution which is rescaled from a monthly to an annual distribution (see Table 6 for descriptive statistics of the annual return distribution),[20] so the parameters are of the form x%, where x is an integer.

In particular, the numerical method searches for integer-valued means and standard deviations for the two normal distributions that can approximate the first four moments of the empirical distribution as closely as possible. Mean, variance, skewness, and kurtosis generally have different dimensions, so the objective function is flexible enough to minimize the weighted

[20] The rescaling technique is described in Appendix A.

relative deviation rather than the absolute deviation. Let w = (w_1, w_2, w_3, w_4) be a vector of strictly positive constants, which serve as weights for the four moments we want to match. The objective function is then:

$$\begin{aligned} \min\ & w_1 \times \left| \frac{\text{theoretical mean-empirical mean}}{\text{empirical mean}} \right| + w_2 \times \left| \frac{\text{theoretical variance-empirical variance}}{\text{empirical variance}} \right| \\ & + w_3 \times \left| \frac{\text{theoretical skewness-empirical skewness}}{\text{empirical skewness}} \right| + w_4 \times \left| \frac{\text{theoretical kurtosis-empirical kurtosis}}{\text{empirical kurtosis}} \right| . \end{aligned} \tag{2}$$

The objective function takes the value of 0 if all four moments can be matched exactly, and positive values otherwise. In this study, all entries in the w-vector equal 1. The approximating parameters for the return distributions of the asset classes are shown in Table 7.

Table 8 shows the first four moments of the empirical return distributions, and compares them with the moments obtained from the mixture of normal method. Obviously, the moments are close and therefore the fit is very good.

The next step is to construct a mixed-asset portfolio consisting of traditional and alternative asset classes. Since the mean-variance approach does not work, we must determine an appropriate objective function. Real-world institutional investors who want to include alternative asset classes in their portfolios are usually endowments, family offices, high net worth individuals, and pension funds. These investors generally seek to obtain higher expected returns than in a money market, but are risk-averse and therefore pay special attention to downside risk.

We specify an objective function of these investors as follows: Let r denote the random return of the portfolio, and r_1 and r_2 denote some benchmark returns. Note that the benchmark returns could be constants or random variables. The investor's objective is to maximize the following function:[21]

$$max \ \ Pr(r > r_1) - \lambda \cdot Pr(r < r_2) \quad (3)$$

In other words, the investor wants to maximize the probability of outperforming some benchmark return, while minimizing the probability of underperforming another benchmark. We assume the first benchmark is some constant, e.g., 10% p.a., or a random return of some other indices such as the S&P 500 as the market return. The second benchmark is usually chosen as 0%, or the risk-free rate or a government bond yield. For this analysis, the first benchmark is defined as a constant 8% p.a., and the second as 0%.

The term λ is a positive constant and represents the trade-off between these two objectives. It is obvious that λ depends on investor risk aversion. The higher λ, the less aggressive the investors, since they weight the second objective more highly and are more concerned about losses than gains. Similar to the relative risk aversion coefficient in canonical utility functions, plausible values of λ lie between 1 and 6. The time horizon for achieving the goal is one year. Therefore, the monthly return distributions of the asset classes must be rescaled to annual return distributions as described earlier.

For the numerical optimization procedure, we consider two constraints: 1) short-selling is not allowed, and 2) the maximum portfolio weight for each asset class is restricted to 30%. Using

[21] This objective function is presented and discussed in Morton, Popova and Popova (2006).

these constraints and the objective function stated above, the optimal portfolio weights for the mixed-asset portfolios are numerically calculated for different parameters for λ.[22]

This method has the advantage over most other frameworks that real investors' preferences can explicitly be considered. They can choose the benchmark which should be outperformed and a second benchmark which they define as a short-fall. Further, investors have to determine their degree of risk aversion only.

5. Results

Figure 2 shows the resulting portfolio weights for the optimal mixed-asset portfolios for λs ranging from 1 to 6 with a step size of 0.5. We can identify three risk aversion regimes: The first regime of low risk aversion ranges from 1 to 2.5, the second of medium risk aversion ranges from 3 to 4, and the third of high risk aversion ranges from 3.5 to 6.

With the first regime, the asset classes' venture capital, buyout, and REITs have the dominant portfolio position. Venture capital and buyout are restricted by 30% when the value of λ is 1, but with increasing risk aversion their weights are reduced to 27% and 25%, respectively. REITs have a constant portfolio weight of 27%. Asset-backed securities, hedge funds, and commodities have minor portfolio weights between 0% and 4% only. U.S. equity and U.S. government bonds also have minor portfolio weights of 5% and 4% when the value of λ is 1, but when risk aversion increases, the portfolio weights increase to 8%.

With the second regime, four asset classes have the dominant portfolio position. In addition to venture capital, buyout, and REITs, U.S. government bonds increases in importance. All four classes have portfolio weights of around 20%. U.S. equity and asset-backed securities have

[22] For computational details, see Morton, Popova and Popova (2006) and the citations.

weights of 8% and between 2% and 8%, respectively. Commodities have a constant portfolio weight of 3%. Hedge funds are not included here in the optimal portfolios.

The high risk aversion regime is comparable to the medium risk aversion regime, with the same four asset classes having a dominant optimal portfolio position. However, the weight of U.S. government bonds increases to the restriction of 30%. The portfolio weights for venture capital, buyout, and REITs decrease slightly when risk aversion increases. When λ reaches the highest degree of risk aversion, the weights for venture capital, buyout, and REITs are 20%, 17%, and 21%, respectively. U.S. equity (7%), hedge funds (2%), and commodities (3%) have minor constant portfolio weights for the degree of risk aversion. Asset-backed securities are not included here in the optimal portfolios.

Our general findings are that U.S. government bonds, venture capital, buyout, and REITs have the largest portfolio weights in the optimal portfolios. The importance of private equity-related asset classes and REITs decreases with increasing risk aversion. The weights of venture capital and buyout decline from 30% when λ equals 1, to 20% and 17%, respectively, when λ equals 6.

The REIT portfolio weights are more stable. When risk aversion is low ($\lambda = 1$), the portfolio weight for REITs equals 26%; when risk aversion is high ($\lambda = 6$), it decreases to 21%. In contrast to the private equity-related asset classes and REITs, U.S. government bonds increase in importance when risk aversion increases. U.S. equity (5%-8%) and commodities (3%-4%) have relatively constant portfolio weights regardless of the degree of risk aversion. Hedge funds are included only in the high and low risk aversion regimes, with portfolio weights of 1%-2%.

In contrast, asset-backed securities in the medium risk aversion portfolios have weights of between 1% and 8%. Our model identifies three risk aversion regimes with relatively constant portfolio weights, so investors do not need to choose the parameter λ as precisely. It is sufficient to determine low, medium, or high risk aversion. This is an advantage of our model, because the risk aversion parameter must be determined a priori.

So far, our approach is based on the assumption that the correlation structure between all these assets remains constant over time. In reality, however, correlations are time-varying and stochastic. It is difficult to include the time-varying correlation structure when planning portfolios, because it is dynamic and renders our numerical optimization very complicated. Therefore, instead of integrating the correlation directly into our portfolio selection problem, we conduct some robustness checks. We check whether our portfolio remains robust against time-varying correlation.

First we draw hundred times from the Wishart distribution,[23] and simulate the new correlation matrix. Then, we run the same optimization procedure as before to determine the new optimal portfolio for three risk aversion regimes: λ = 1, 3, and 6. If the new portfolio does not deviate greatly from the initial portfolio, we can conclude that our initial portfolio remains stable and robust against time-varying correlations.

As the results of the robustness tests in Table 9 show, our initial portfolio is quite stable, especially the high risk aversion one. In particular, the principle portfolio components remain the same for many simulated correlation matrices. In some cases, the new portfolio is exactly the same; in others, the weights in some assets have minor changes.

[23] For further details, see, e.g., Zhang (2006).

6. Conclusion

Return distributions of alternative and traditional asset classes often exhibit non-normal properties. Furthermore, some return time series of alternative investments suffer from several biases. To capture their risk-return profiles adequately, we first correct for these biases. Our next step is to consider the higher moments of the return distributions in the strategic asset allocation model, because the variance does not capture all the sources of risk.

We use a model here that is flexible enough to integrate the higher moments and real investor preferences. The resulting portfolio weights for three different risk aversion regimes – low, medium, and high – are presented. Alternative investments are found to play an outstanding role in efficient portfolios, regardless of the level of risk aversion. In particular, the alternative investment asset classes buyout, REITs, and venture capital have substantial portfolio weights under all regimes.

In contrast, U.S. government bonds are underrepresented in efficient portfolios in regimes with low degrees of risk aversion. However, when risk aversion increases, the portfolio weight also increases up to the maximum of 30%. This means that U.S. government bonds are most valuable when the focus is on downside risk protection rather than enhancing expected returns. U.S. equity, as the second traditional asset class, is represented in all efficient portfolios with a relatively constant portfolio weight of about 7%, regardless of the level of risk aversion.

Commodities exhibit similar behaviour to U.S. equity, but the portfolio weight of about 3% is lower. Hedge funds and asset-backed securities are not represented in all risk aversion regimes. Hedge funds are represented only in low and high regimes, with minor portfolio weights of about 2%; asset-backed securities are represented in medium risk aversion regimes, with about 4%.

In conclusion, alternative investments are important for the strategic asset allocation of institutional investors such as endowments, family offices, pension funds, and high net worth individuals who have sufficient time horizons and investment capital. However, not all alternative investment classes are of equal importance. Alternative investments are not appropriate as substitutes for traditional asset classes, and may be better as complements for achieving the desired risk-return profiles.

Table 1
Data Descriptions

This table reports the proxy indices for each asset class. The frequency, inception dates, end date, and additional information sources are given for the proxy time series.

Asset Class	Proxy Index	Frequency	Inception Date	End Date	Additional Information
U.S. Equity	S&P 500 Composite - Total Return Index	Monthly	Jan 98	Jul 06	http://www2.standardandpoors.com
U.S. Government Bonds	JPM United States Govt. Bond - Total Return Index	Monthly	Jan 98	Jul 06	http://www.datastream.com
Asset-Backed Securities	IBOXX Coll. ABS - Total Return Index	Monthly	Jan 98	Jul 06	http://www.datastream.com
Buyout	CepreX U.S. Buyout	Monthly	Jan 98	Jul 06	http://www.cepres.de
Commodities	S&P GSCI Commodity Total Return Index	Monthly	Jan 98	Jul 06	http://www.datastream.com
Hedge Funds	Credit Suisse/Tremont Hedge Fund Index	Monthly	Jan 98	Jul 06	http://www.hedgeindex.com
Real Estate Investment Trusts	FTSE/NAREIT Equity REITS - Total Return Index	Monthly	Jan 98	Jul 06	http://www.nareit.com
Venture Capital	CepreX U.S. Venture Capital	Monthly	Jan 98	Jul 06	http://www.cepres.de

Table 2
Descriptive Statistics of the (Partially Biased) Monthly Return Distributions

This table reports the arithmetic mean, median, maximum, minimum, standard deviation, skewness, and kurtosis of the monthly return distributions from January 1998 through July 2006. The descriptive statistics are calculated on the raw return time series obtained from the data vendors.

	JPM Gov. Bonds	S&P GSCI	CS/T Hedge Fund Index	IBOXX	NAREIT	S&P 500	CepreX U.S. Buyout	CepreX U.S. Venture Capital
Mean	0.0045	0.0101	0.0072	0.0057	0.0110	0.0050	0.0281	0.0197
Median	0.0050	0.0075	0.0075	0.0064	0.0156	0.0084	0.0167	0.0104
Maximum	0.0356	0.1579	0.0853	0.0498	0.0949	0.0978	0.5694	0.2236
Minimum	-0.0449	-0.1392	-0.0755	-0.0310	-0.1458	-0.1446	-0.2116	-0.2487
Std. Dev.	0.0142	0.0638	0.0202	0.0135	0.0412	0.0450	0.1261	0.0888
Skewness	-0.5176	0.0545	0.0189	0.0346	-0.6505	-0.4786	1.6333	0.1233
Kurtosis	3.6741	2.5796	7.7529	3.3737	4.0580	3.4796	8.2413	3.7849

Table 3
First-Order Autocorrelation of the Asset Classes

This table reports the coefficient of the first-order autocorrelation (AR(1)) for monthly returns. The Ljung-Box test [Ljung and Box (1978)] is used to test for significance.

Asset Class	AR(1)
S&P 500	0.008
ABS	-0.056
CS/T Hedge Fund Index	0.207**
JPM Gov. Bonds	0.008
CepreX U.S. Venture Capital	-0.330***
CepreX U.S. Buyout	-0.411***
S&P GSCI	0.039
NAREIT	-0.064

***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively, based on monthly returns.

Table 4
Descriptive Statistics of the (Unbiased) Monthly Return Distributions

This table reports the arithmetic mean, median, maximum, minimum, standard deviation, skewness, and kurtosis of the monthly return distributions from January 1998 through July 2006. The return time series of the CS/T Hedge Fund Index is corrected for survivorship bias. Every monthly return is reduced by 2.5% to account for annual survivorship bias. The return time series of the S&P 500 is increased by 5% p.a. Furthermore, the return time series of the CS/T Hedge Fund Index, CepreX U.S. Buyout, and CepreX U.S. Venture Capital are adjusted for first-order autocorrelation following Geltner (1991) method. We use the Jarque-Bera test [Jarque and Bera (1980)] to test for the assumption of normally distributed monthly returns.

	JPM Gov. Bonds	S&P GSCI	CS/T Hedge Fund Index	IBOXX	NAREIT	S&P 500	CepreX U.S. Buyout	CepreX U.S. Venture Capital
Mean	0.0045	0.0101	0.0055	0.0057	0.0110	0.0086	0.0280	0.0201
Median	0.0050	0.0075	0.0039	0.0064	0.0156	0.0084	0.0175	0.0111
Maximum	0.0356	0.1579	0.0916	0.0498	0.0949	0.0978	0.4213	0.1823
Minimum	-0.0449	-0.1392	-0.0985	-0.0310	-0.1458	-0.1446	-0.1530	-0.1528
Std. Dev.	0.0142	0.0638	0.0250	0.0135	0.0412	0.0450	0.0879	0.0649
Skewness	-0.5176	0.0545	-0.0452	0.0346	-0.6505	-0.4786	1.8119	0.2269
Kurtosis	3.6741	2.5796	7.2968	3.3737	4.0580	3.4796	8.9188	3.5350
Jarque-Bera	6.49**	0.80	78.50***	0.61	11.95***	4.87*	204.70***	2.09

***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively, based on monthly returns.

Table 5
Correlation Matrix

This table reports the correlation between the asset classes based on monthly returns.

	JPM Gov. Bonds	S&P GSCI	CS/T Hedge Fund Index	ABS	NAREIT	S&P 500	CepreX U.S. Buyout	CepreX U.S. Venture Capital
JPM Gov. Bonds	1.000	0.027	-0.131	0.628	-0.032	-0.269	-0.097	-0.152
S&P GSCI	0.027	1.000	0.025	-0.008	0.145	0.014	0.142	-0.095
CS/T Hedge Fund	-0.131	0.025	1.000	-0.054	0.201	0.106	-0.116	-0.026
ABS	0.628	-0.008	-0.054	1.000	0.028	-0.126	-0.054	0.039
NAREIT	-0.032	0.145	0.201	0.028	1.000	0.106	-0.052	-0.142
S&P 500	-0.269	0.014	0.106	-0.126	0.106	1.000	0.223	-0.003
Buyout	-0.097	0.142	-0.116	-0.054	-0.052	0.223	1.000	-0.002
Venture Capital	-0.152	-0.095	-0.026	0.039	-0.142	-0.003	-0.002	1.000

Table 6
Descriptive Statistics of the Rescaled Annual Return Distribution

This table reports the arithmetic mean, median, maximum, minimum, standard deviation, skewness, and kurtosis of the annual return distribution from January 1998 through July 2006. The median, maximum, and minimum are obtained from rolling annual returns. The first four central moments are rescaled from monthly to annual data, because otherwise the model calibration would be based on eight observations. See Appendix A for computational details.

	JPM Gov. Bonds	S&P GSCI	CS/T Hedge Fund Index	IBOXX	NAREIT	S&P 500	CepreX U.S. Buyout	CepreX U.S. Venture Capital
Mean	0.055	0.122	0.066	0.068	0.132	-0.103	0.336	0.241
Median	0.049	0.195	0.062	0.080	0.174	0.087	0.301	0.089
Maximum	-0.034	-0.357	-0.090	-0.022	-0.211	-0.266	-0.205	-0.343
Minimum	0.154	0.722	0.319	0.136	0.526	0.398	1.499	1.876
Std. Dev.	0.049	0.221	0.087	0.047	0.143	0.156	0.305	0.225
Skewness	-0.149	0.016	-0.013	0.010	-0.188	-0.138	0.523	0.065
Kurtosis	3.056	2.965	3.358	3.031	3.088	3.040	3.493	3.045

Table 7
Moments of the Normally Distributed Auxiliary Distributions

This table shows the mean and standard deviation of the two auxiliary distributions, as well as the weighting factor for all asset classes. The values in the w-vector are equal to 1.

Auxiliary Distributions	Distribution 1		Distribution 2	
Weighting Factor	0.2		0.8	
	Mean	Standard Deviation	Mean	Standard Deviation
S&P 500	2%	16%	13%	15%
ABS	8%	4%	6%	4%
CS/T Hedge Fund Index	7%	8%	6%	9%
JPM Gov. Bonds	1%	4%	6%	4%
CepreX U.S. Venture Capital	34%	24%	21%	22%
CepreX U.S. Buyout	48%	39%	30%	27%
S&P GSCI	26%	19%	9%	20%
NAREIT	20%	9%	11%	15%

Table 8
Comparison of the Moments of Empirical and Approximated Distributions

This table shows the first four moments of the empirical and approximated distribution. The number on the left is the theoretical moment in the approximated distribution; the number in parentheses is the empirical moment.

	Mean	Standard Deviation	Skewness	Kurtosis
JPM Gov. Bonds	0.050 (0.055)	0.045 (0.049)	0.020 (-0.149)	3.010 (3.056)
S&P GSCI	0.124 (0.122)	0.209 (0.221)	0.017 (0.016)	2.973 (2.965)
CS/T Hedge Fund Index	0.062 (0.066)	0.088 (0.087)	-0.012 (-0.013)	3.021 (3.358)
ABS	0.064 (0.068)	0.041 (0.047)	0.011 (0.010)	3.000 (3.031)
NAREIT	0.128 (0.132)	0.145 (0.143)	-0.183 (-0.188)	3.075 (3.088)
S&P 500	0.108 (-0.103)	0.158 (0.156)	-0.073 (-0.138)	3.043 (3.040)
CepreX U.S. Buyout	0.336 (0.336)	0.306 (0.305)	0.257 (0.523)	3.510 (3.493)
CepreX U.S. Venture Capital	0.236 (0.241)	0.230 (0.225)	0.064 (0.065)	3.047 (3.045)

Table 9
Robustness Check – Time-Varying Correlation Matrix

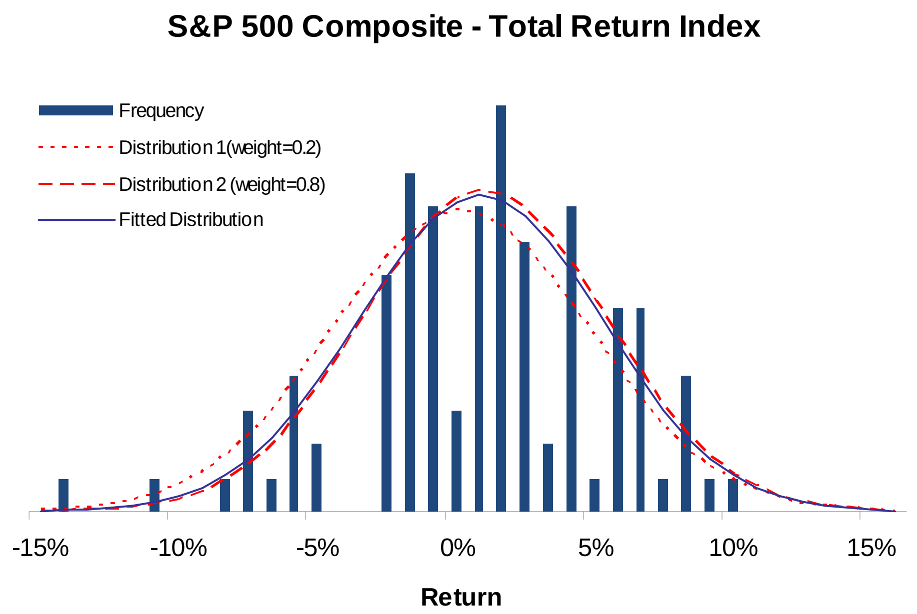
This table shows the optimal portfolio when the variance of covariance is drawn from a Wishart distribution. The numbers on the left are the ranges of the portfolio weights for the random sample for 100 runs. The number in parentheses is the portfolio weight for the original correlation matrix.

	$\lambda=1$	$\lambda=3$	$\lambda=6$
S&P 500	3%-6% (5%)	5%-12% (8%)	5%-10% (7%)
ABS	0%-1% (0%)	0%-3% (2%)	0%-2% (0%)
CS/T Hedge Fund Index	0%-5% (2%)	0% (0%)	0%-3% (2%)
JPM Gov. Bonds	3%-7% (4%)	9%-12% (10%)	28%-30% (30%)
CepreX U.S. Venture Capital	30% (30%)	25%-30% (27%)	19%-23% (20%)
CepreX U.S. Buyout	25%-30% (29%)	21%-30% (23%)	14%-19% (17%)
S&P GSCI	2%-5% (4%)	1%-5% (3%)	1%-5% (3%)
NAREIT	22%-28% (26%)	24%-30% (27%)	18%-23% (21%)

Figure 1

Return Histograms and Fitted Return Distributions for the Asset Classes

The figure shows the monthly return histogram of the eight asset classes and the corresponding fitted return distribution for asset classes. The fitted return distribution is composed of two auxiliary distributions – distribution 1 and distribution 2 – that are weighted with factors 0.2 and 0.8, respectively. The y-axis shows the frequency for the histogram and the probability density for the continuous distributions.



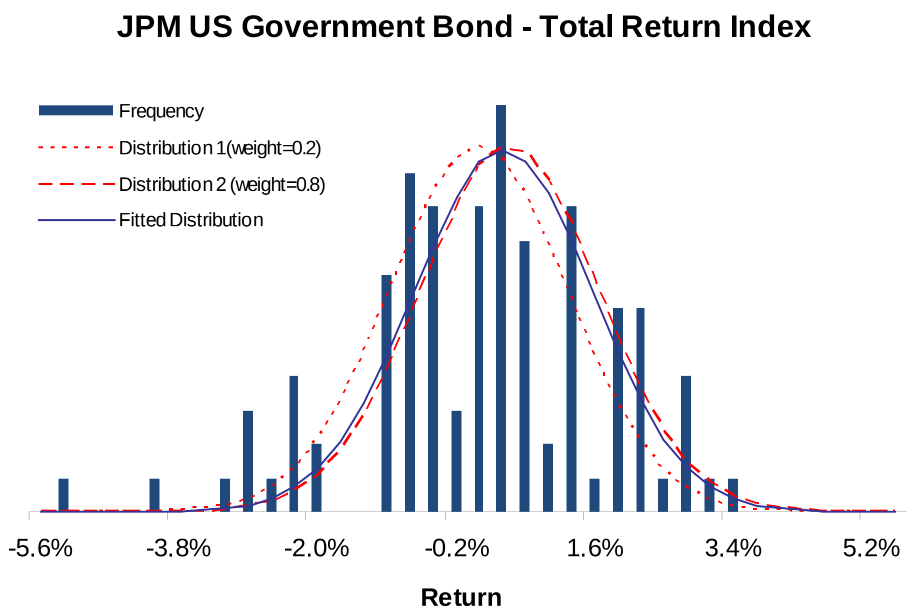
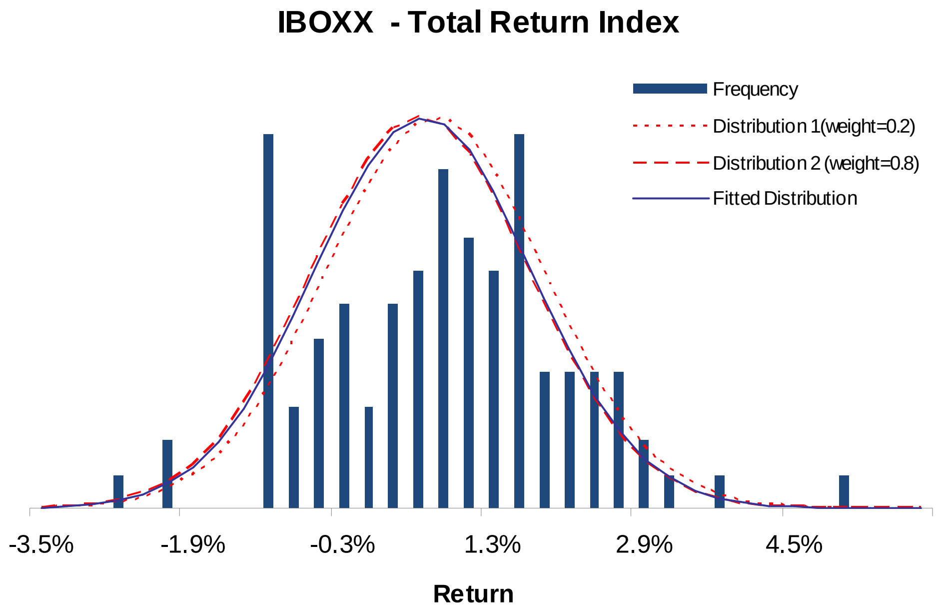


Figure 1—*Continued*



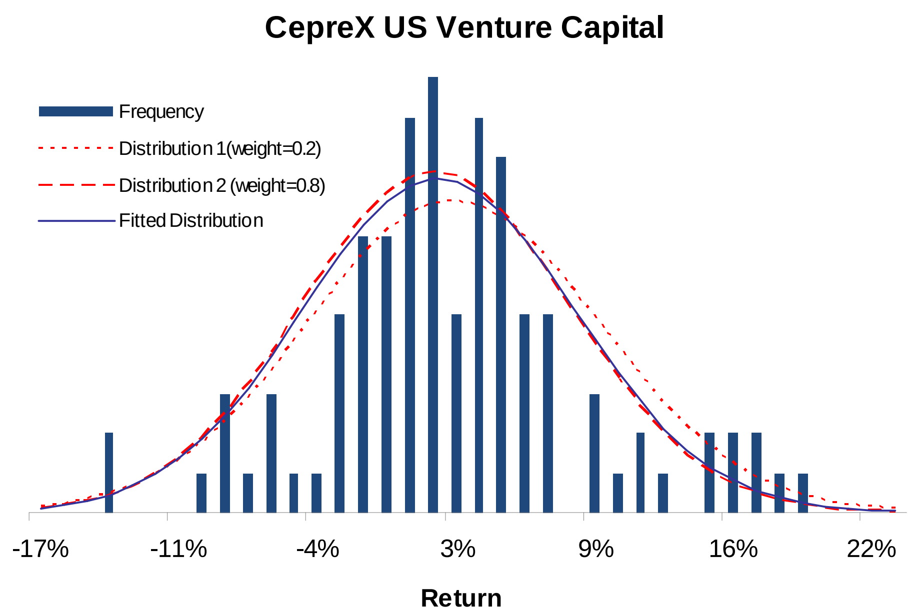
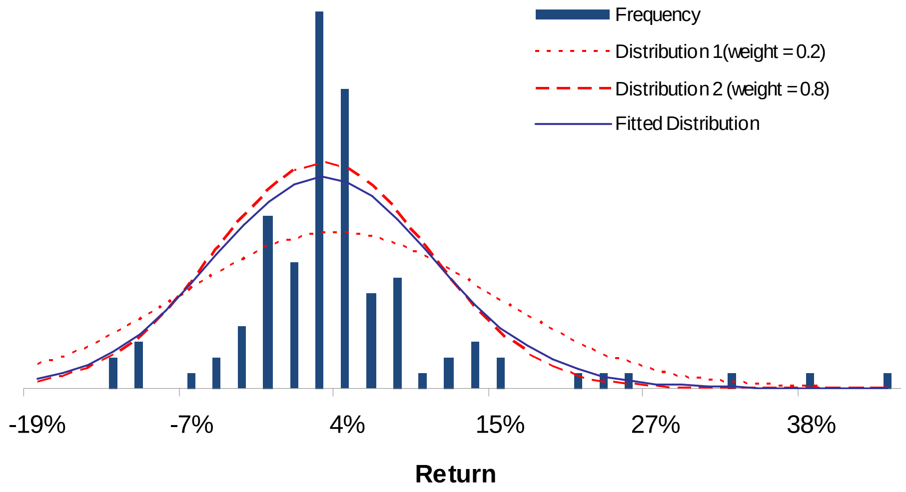


Figure 1—*Continued*

CepreX US Buyout



S&P GSCI Commodity Total Return Index

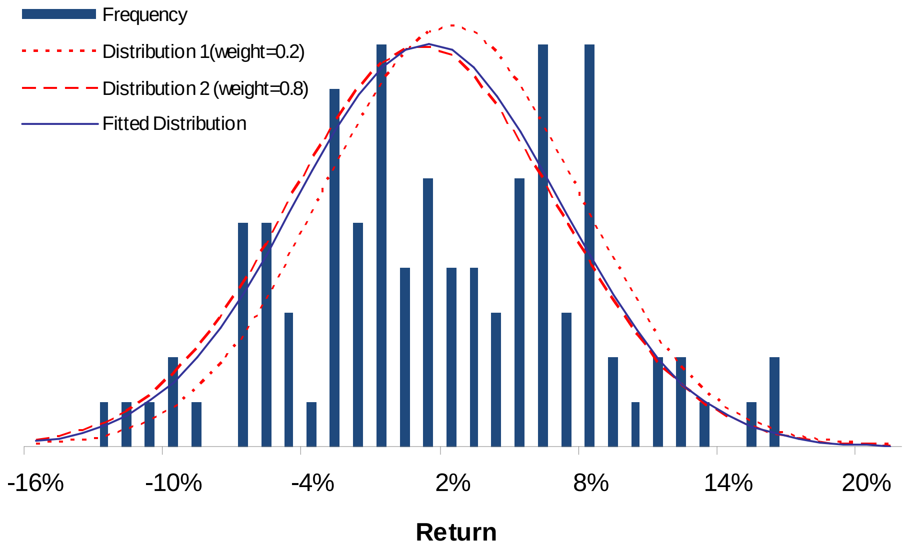
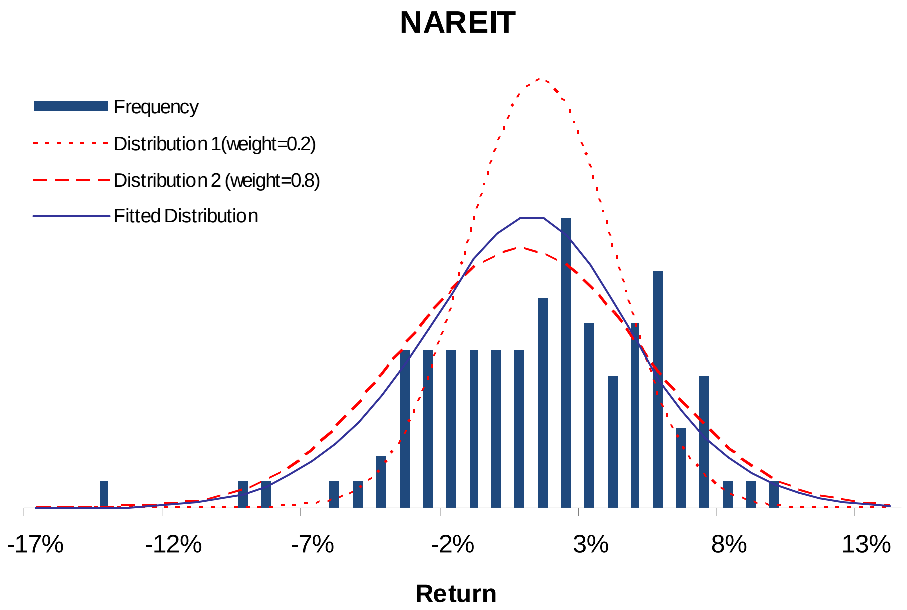


Figure 1—*Continued*



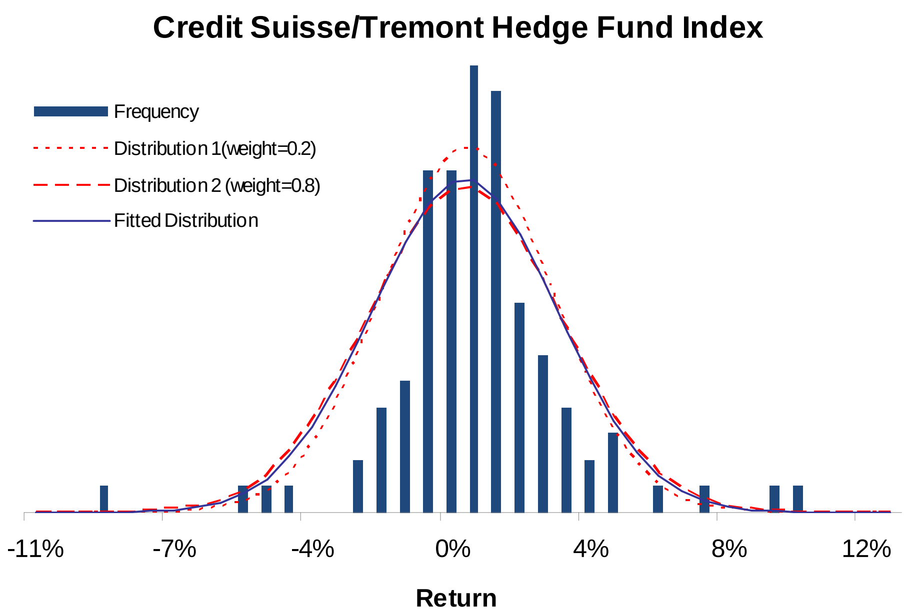
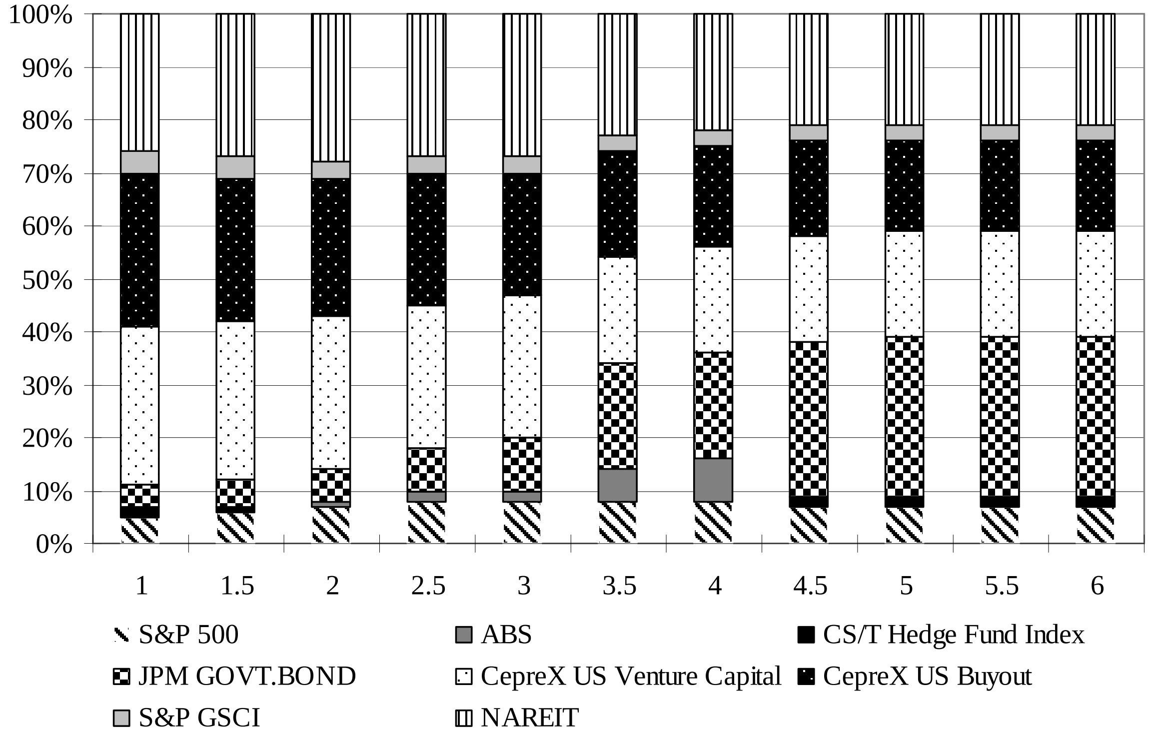


Figure 2
Optimal Portfolio Weights

This figure displays the relationship between the risk aversion factor λ and the corresponding optimal portfolio weights for the asset classes.



Appendix A

The moments of a monthly return distribution can be rescaled to an annual return distribution as follows. Let r_i denote the monthly return, i and R denote the annual return.

It is obvious that

$$R = \sum_{i=1}^{12} r_i \ .$$

Assume r_is are iid. Let $E[r_i] = \bar{r}$, $Var(r_i) = \sigma_r^2$, $E[R_i] = \bar{R}$, and $Var(R_i) = \sigma_R^2$. It is well known that

$$\bar{R} = 12\bar{r}$$

$$\sigma_R = \sqrt{12}\sigma_r .$$

The skewness of the annual return distribution can be derived from the monthly distribution as follows:

$$Skew(R) = \frac{Skew(r_i)}{\sqrt{12}} .$$

The kurtosis of the annual return distribution can be derived from the monthly distribution as follows:

$$Kurt(R) = \frac{Kurt(r_i)}{12} + \frac{11}{4}.$$

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